VOLUME 86 NO. ST12
DECEMBER 1960
PART 1

JOURNAL of the

Structural Division

PROCEEDINGS OF THE

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OF CIVIL ENGINEERS

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The index for 1959 was published as ASCE Publication 1960-10 (list price $2.00); indexes for previous years are also available.

Journal of the

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

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Journal of the
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STRESS ANALYSIS OF RIGID FRAME BRIDGES WITH INCLINED LEGS

By Shih-yuan Cheng[1]

SYNOPSIS

Methods of direct moment distribution applied to the rigid frame bridges with inclined legs are introduced in this paper. Formulas are derived for the anti-symmetric loading conditions. By using these formulas with the aid of the tables that are provided, a simplified method for influence line analysis is introduced.

INTRODUCTION

The rigid frame bridges discussed in this paper consist of the four types shown in Fig. 1. All the structures are symmetric and the members are of uniform section. Types C and D are in common use and will be fully discussed here. A direct method of moment distribution will be introduced that can also be applied to the structures mentioned. The proposed method contains only one distribution and carry-over, and in addition, no successive approximations or sidesway corrections are needed.

Because any loadings can be considered as the combination of the symmetric and anti-symmetric ones, the procedure of influence line analysis is much simplified. A series of unitless C-values are provided with which the computation work can be considerably reduced.

Note.—Discussion open until May 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 12, November, 1960.

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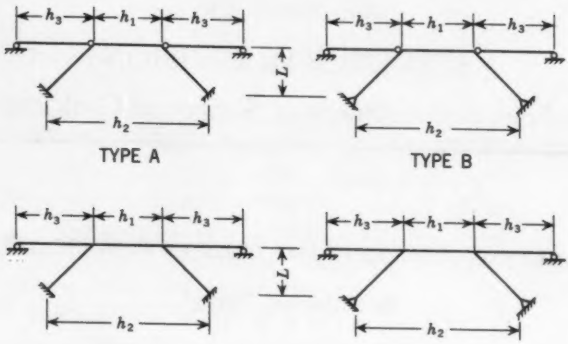


FIG. 1

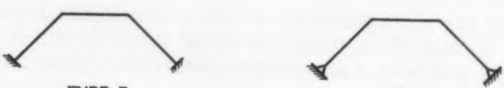
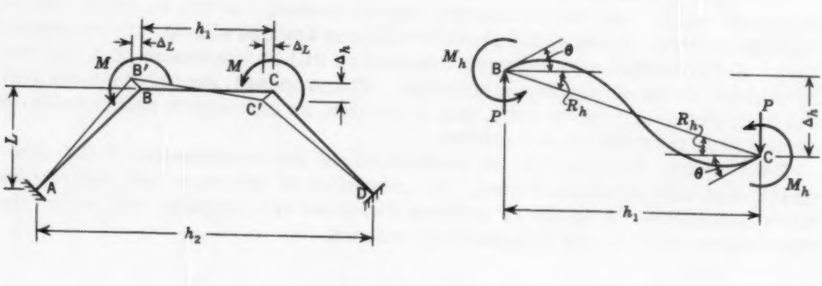


FIG. 2



FIGS. 3 AND 4

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in Appendix II.

STRESS ANALYSIS FOR ANTI-SYMMETRIC LOADINGS CONDITION

We first consider the rigid frame of type A. Its stresses analysis is like that of the type E frame (Fig. 2). The formulas for stiffness, carry-over factor, and fixed-end moment are derived as follows.

Stiffness and Carry-over Factor.—As shown in Fig. 3, if we apply the same external moments M both at joints B and C simultaneously, then these joints will rotate and translate. Assume their final positions after rotation and translation be B' and C', and let Δ_L be the horizontal displacement of B and C, Δ_h the relative deflection between them, then we may obtain by geometry the relation

$$\frac{\Delta_h}{h_1} = - \frac{h_2 - h_1}{h_1} \frac{\Delta_L}{L} \qquad (1)$$

or

$$R_h = -\alpha R_L \qquad (2)$$

Because the frame is a symmetric structure, and the applied moments are anti-symmetric loadings, joints B and C will rotate through the same angles both in magnitude and direction. Taking member BC as a free body then, by the theorem of slope deflection, the resistant moments and shears at both ends of member BC are

$$\begin{aligned} M_h &= 6\ E\ K_h\left(\theta - R_h\right) \\ &= 6\ E\ K_h\left(\theta + \alpha\ R_L\right) \qquad (3) \end{aligned}$$

$$P = \frac{12\ E\ K_h}{h_1}\left(\theta + \alpha\ R_L\right) \qquad (4)$$

Taking member AB as a free body again, because of symmetric structure and anti-symmetric loadings, member AB and CD will not restrain each other when they are rotating and displacing. In other words, the axial force acting at either end of member BC will be zero. Therefore, we may conclude that there is only a moment M and a vertical force P that act at end B of member AB. Under the action of such a force system, end B will rotate through an angle θ and translate by an amount of Δ_L. According to moment area theorem, the relationships between the deformations and this force system are

$$\frac{M_L}{E\ K_L} - 3\ \frac{K_h}{K_L}\left(\alpha^2 R_L + \alpha\theta\right) = \theta \qquad (5)$$

and

$$\frac{M_L}{E\ K_L} - 2\ \frac{K_h}{K_L}\left(\alpha^2 R_L + \alpha\theta\right) = R_L \qquad (6)$$

Solving Eqs. 5 and 6 gives

$$M_L = \frac{4\,n\,\alpha^2 + 6\,n\,\alpha + 2}{n\,\alpha^2 + 2}\,E\,K_L\theta \quad \text{(7a)}$$

When $\theta = 1$, the value of M_L will be the stiffness of member AB at end B. That is,

$$S\,a_L = \frac{4\,n\,\alpha^2 + 6\,n\,\alpha + 2}{n\,\alpha^2 + 2}\,E\,K_L \quad \text{(7b)}$$

from Eqs. 6 and 7, we can also obtain another relationship between R_L and θ, namely

$$R_L = \frac{1 + n\,\alpha}{n\,\alpha^2 + 2}\,\theta \quad \text{(8)}$$

substituting this value in Eq. 2 gives

$$M_h = \frac{\alpha + 2}{n\,\alpha^2 + 2}\,6\,E\,K_h\theta \quad \text{(9a)}$$

Similarly, the stiffness of member BC at end B is

$$S\,a_h = \frac{\alpha + 2}{n\,\alpha^2 + 2}\,6\,E\,K_h \quad \text{(9b)}$$

In Fig. 5 $\Sigma M = 0$ and

$$C\,a_{ba}\,M_L = P\,\frac{h_2 - h_1}{2} - M_L \quad \text{(10)}$$

Solving Eqs. 4, 5 and 6 gives

$$C\,a_{ba} = \frac{2\,n\,\alpha^2 + 6\,n\,\alpha - 2}{4\,n\,\alpha^2 + 6\,n\,\alpha + 2} \quad \text{(11)}$$

Fixed End Moment.—In Fig. 7, the frame is subjected to a horizontal force P acting at joint B. If we assume that the joints B and C are kept from rotation, then the fixed end moments may be expressed as

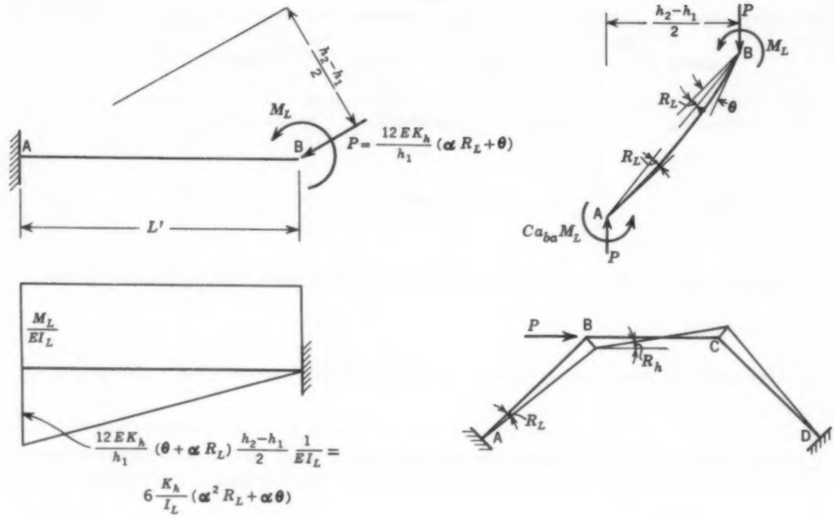
$$M_{F_L} = -\,6\,E\,K_L\,R_L \quad \text{(12a)}$$

and

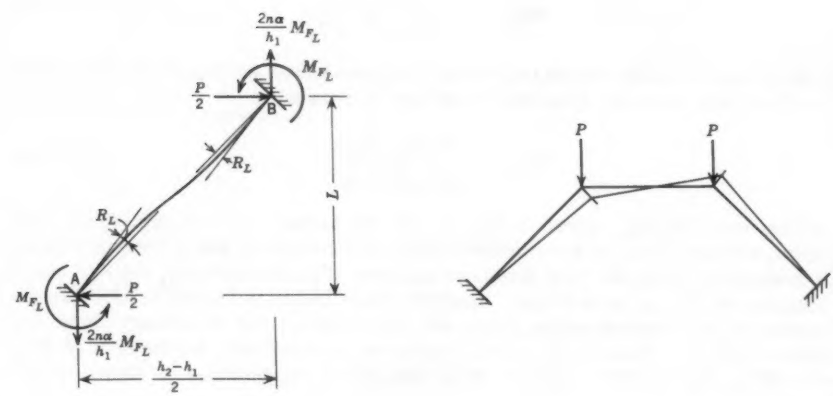
$$M_{F_h} = -\,6\,E\,K_h\,R_h \quad \text{(12b)}$$

Thus,

$$M_{F_h} = -\,n\,\alpha\,M_{F_L} \quad \text{(13)}$$



FIGS. 5, 6 AND 7



FIGS. 8 AND 9

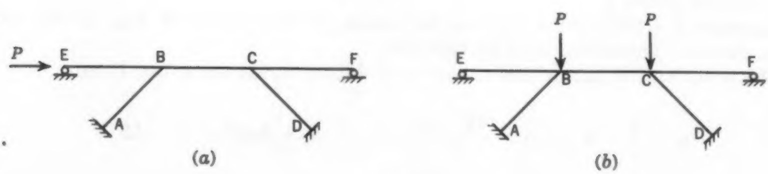


FIG. 10

TABLE 1

Type	S_{ba}		S_{bc}		S_{be}
	absolute	relative	absolute	relative	absolute
(1)	(2)	(3)	(4)	(5)	(6)
E	$\frac{1}{A}\ 4n\alpha^2+6n\alpha+2\ EK_{ba}$	$4n\alpha^2+6n\alpha+2$	$\frac{1}{A}(\alpha+2)6EK_{bc}$	$(\alpha+2)6n$	
F	$\frac{1}{A}\alpha(\alpha+1)6EK_{bc}$	α	$\frac{1}{A}(\alpha+1)6EK_{bc}$	1	
C	$\frac{1}{A}\left[2n_1\alpha(2\alpha+3)-\frac{1}{2}n_2\beta(3-\beta)+2\right]EK_{ba}$	$2n_1\alpha(2\alpha+3)-\frac{1}{2}n_2\beta(3-\beta)+2$	$\frac{1}{A}\left[\alpha+\frac{1}{8}n_2\beta(2\alpha+\beta)+2\right]6EK_{bc}$	$\left[\alpha+\frac{1}{8}n_2\beta(2\alpha+\beta)+2\right]6n_1$	$\frac{1}{A}\left[\frac{1}{2}n_1\alpha(2\alpha+\beta)-\frac{1}{2}\beta+2\right]3EK_{be}$
D	$\frac{1}{A}\left[n_1\alpha(\alpha+1)-\frac{1}{8}n_2\beta(2-\beta)\right]6EK_{ba}$	$6n_1\alpha(\alpha+1)-\frac{3}{4}n_2\beta(2-\beta)$	$\frac{1}{A}\left[\alpha+\frac{1}{4}n_2\beta(2\alpha+\beta)+1\right]6EK_{bc}$	$\left[\alpha+\frac{1}{4}n_2\beta(2\alpha+\beta)+1\right]6n_1$	$\left[\frac{1}{2}n_1\alpha(2\alpha+\beta)-\frac{1}{2}\beta+2\right]3n_2$

[a]Absolute stiffness is that moment, that when applied at the member end will cause unit rotation there. The relative stiffness is the rela-
Figs. 7, 9, 10.

Take member AB as a free body (Fig. 8) and, by equilibrium of force system, we get

$$M_{F_L} = + \frac{P\ L}{2\left(n\ \alpha^2 + 2\right)} \qquad (14a)$$

If the frame is subjected to two vertical forces shown in Fig. 9, we can obtain the fixed end moments from the same line of reasoning

$$M_{F_L} = \frac{P\left(h_2 - h_1\right)}{2\left(n\ \alpha^2 + 2\right)} \qquad (14b)$$

and the value of M_{F_h} given in Eq. 13. If the forces are not applied at the joints, we may first, by keeping the joint from translation, apply the conventional method to obtain the first fixed end moment M'_F. Secondarily, the end reactions due to M'_F may be found. Applying these reaction forces in opposite direction at the corresponding joints we again obtain the secondary fixed end moment M''_F. Finally, the actual fixed end moment will be the sum of M'_F and M''_F, that is $M_F = M'_F + M''_F$ (the detail computation is shown in example 3).

For the frames of types B, C, and D the equations for stiffness, carry-over factor, and fixed end moment are similarly derived. All these values are tabulated in Table 1.

Example 1.—Determine the end moments on the frame of Fig. 12 for the loads shown. The solution is as follows:

The distribution factor and carry-over factor are determined from

$$n = \frac{2}{1} = 2 \qquad \alpha = \frac{12 - 4}{4} = 2 \qquad A = 2x2^2 + 2 = 10$$

relative (7)	Ca_{ba} (8)	$M_{F_{ba}}$ or $M_{F_{ab}}$ Vertical Loads (9)	Horizontal Loads (10)	$M_{F_{bc}}$ (11)	$M_{F_{be}}$ (12)	Remarks (13)
	$\frac{1}{Sr_{ba}}(2n\alpha^2+6n\alpha-2)$	$+\frac{P(h_2-h_1)}{2A}$	$+\frac{PL}{2A}$	$-n\alpha M_{F_{ba}}$		$A=n\alpha^2+2$ $R_L=\frac{1-n\alpha}{A}\theta$
	0	$+\frac{P(h_2-h_1)}{2A}$	$+\frac{PL}{2A}$	$-2n\alpha M_{F_{ba}}$		$A=2n\alpha^2+1$ $R_L=\frac{1-2n\alpha}{A}\theta$
$\frac{1}{A}\left[n\alpha(2\alpha+\beta)-\frac{1}{2}\beta+1\right]3EK_{be}$	$\frac{1}{Sr_{ba}}\left[2n_1\alpha(\alpha+3)-\frac{1}{4}n_2\beta(6-\beta)-2\right]$	$+\frac{P(h_2-h_1)}{2A}$	$+\frac{PL}{2A}$	$-n_1\alpha M_{F_{ba}}$	$+\frac{1}{4}n_2\beta M_{F_{ba}}$	$A=n_1\alpha^2+\frac{1}{8}n_2\beta^2+2$ $R_L=\frac{1}{A}\left(1-n_1\alpha+\frac{1}{4}n_2\beta\right)\theta$
$\left[n_1\alpha(2\alpha+\beta)-\frac{1}{2}\beta+1\right]3n_2$	0	$+\frac{P(h_2-h_1)}{2A}$	$+\frac{PL}{2A}$	$-2n_1\alpha M_{F_{ba}}$	$+\frac{1}{2}n_2\beta M_{F_{ba}}$	$A=2n_1\alpha^2+\frac{1}{4}n_2\beta^2+1$ $R_L=\frac{1}{A}\left(1-2n_1\alpha+\frac{1}{2}n_2\beta\right)\theta$

tive value of stiffness amounts for several members meeting at a joint. For moment distribution the relative stiffness only is used. See

$$Sr_{ba} = 4\ 2\ 2^2 + 6\ 2\ 2 + 2 = 58 \qquad 0.547$$

$$Sr_{bc} = (2+2)\ 6\ 2 = \qquad 48 \qquad 0.453$$

$$106 \qquad 1.000$$

$$Ca_{ba} = \frac{2\ 2\ 2^2 + 6\ 2\ 2 - 2}{58} = 0.655$$

The fixed end moments are

$$M_{F_{ab}} = \frac{5\ 4}{2\ 10} = 1.00 \text{ ft-kip}$$

$$M_{F_{bc}} = -\ 2\ 2\ 1 = -4.00 \text{ ft-kip}$$

The moment distribution is determined as

ba	bc
0.547	0.453
+1.00	-4.00
+1.64	+1.36
+2.64	-2.64

0.655

+1.00
+1.07
+2.07

Thus M_{ba} = +2.64 kip-ft and M_{ab} = +2.07 kip-ft.

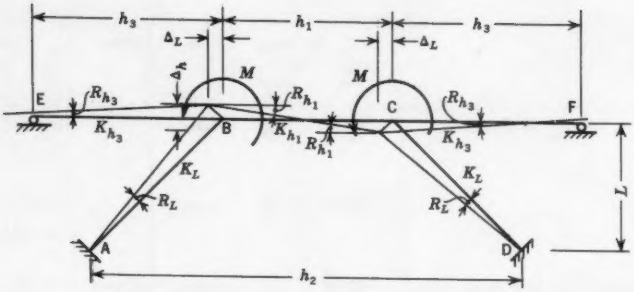
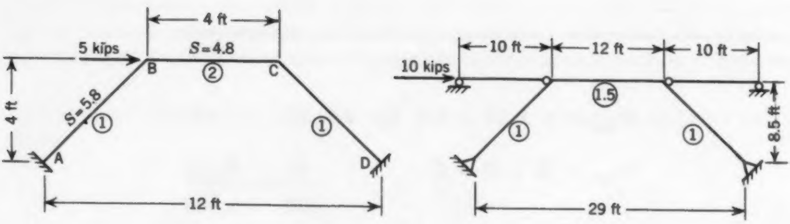


FIG. 11



FIGS. 12 AND 13

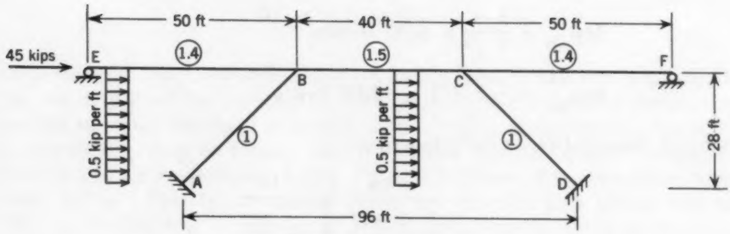


FIG. 14

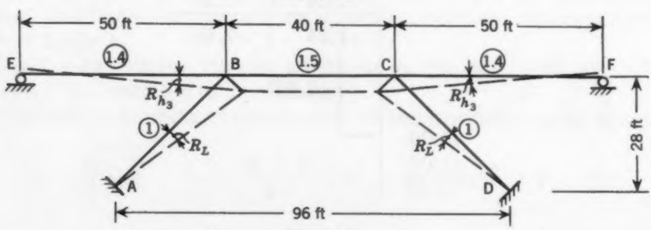


FIG. 15

Example 2.—Determine the joint moments caused in the structure of Fig. 13 by the 10 kip horizontal load shown. The solution is as follows:

The distribution factor and carry-over factor are determined from

$$n = 1.5 \qquad \alpha = \frac{29 - 12}{12} = 1.416$$

$$A = 2\ 1.5\ 1.416^2 + 1 = 7.01$$

$$\begin{array}{lll} Sr_{ba} = & 1.416 & 0.856 \\ Sr_{bc} = & \underline{1.000} & \underline{0.414} \\ & 2.416 & 1.000 \end{array}$$

The fixed end moments are

$$M_{F_{ba}} = + \frac{10\ 8.5}{2\ 7.01} = +6.05 \text{ ft-kip}$$

$$M_{F_{bc}} = -\ 2\ 1.5\ 1.416\ 6.05 = -25.78 \text{ ft-kip}$$

The moment distribution is as follows:

ab	ba	bc
	0.586	0.414
	+ 6.05	-25.78
	+11.57	+ 8.16
	+17.62	-17.62
0		

Thus $M_{ba} = -M_{bc} = 17.62$ ft-kip.

Example 3.—Determine the end moments of the structure of Fig. 14 by the seismic loads shown in this figure. The solution is as follows:

The distribution factor and carry-over factor are determined from

$$n_1 = 1.5 \qquad n_2 = 1.4 \qquad \alpha = \frac{96-40}{40} = 1.4 \qquad \beta = \frac{96-40}{50} = 1.12$$

$$A = 1.5\ 1.4 + \frac{1}{8}\ 1.4\ 1.12 + 2 = 5.159$$

$$Sr_{ba} = 2\ 1.5\ 1.4\ (2\ 1.4{+}3) - \frac{1}{2}\ 1.4\ 1.12\ (3{-}1.12) + 2 = 24.88 \qquad 0.290$$

$$Sr_{bc} = \left[1.4 + \frac{1}{8}\ 1.4\ 1.12\ (2\ 1.4{+}1.12) + 2\right]\ 6\ 1.5 = \qquad 37.50 \qquad 0.438$$

$$Sr_{be} = \left[\frac{1}{2}\ 1.5\ 1.4\ (2\ 1.4{+}1.12) - \frac{1}{4}\ 1.12{+}2\right]\ 3\ 1.4 = \qquad \underline{23.28} \qquad \underline{0.272}$$

$$85.66 \qquad 1.000$$

$$Ca_{ba} = \frac{1}{24.88}\left[2\ 1.5\ 1.4\ (1.4+3) - \frac{1}{4}\ 1.4\ 1.12\ (6-1.12) - 2\right]$$

$$= 0.585$$

The fixed end moments are

$$M'_{F_{ba}} = -\ M'_{F_{ab}}$$

$$= -\ \frac{1}{12}\ 0.5\ 28^2 = -32.65 \text{ ft-kip}$$

$$R = \frac{1}{2}\ 28\ 0.5 = 7^k$$

$$M''_{F_{ba}} = +\ \frac{(45 + 2\ 7)\ 28}{2\ 5.159} = 160.0 \text{ ft-kip}$$

$$M''_{F_{ab}} = -160.0 \text{ ft-kip}$$

$$M_{F_{ba}} = -32.65 + 160.0 = 127.35 \text{ ft-kip}$$

$$M_{F_{ab}} = 32.65 + 160.0 = 192.65 \text{ ft-kip}$$

$$M_{F_{bc}} = -1.5\ 1.4\ 160.0 = -336 \text{ ft-kip}$$

and

$$M_{F_{be}} = \frac{1}{4}\ 1.4\ 1.12\ 160.0 = 62.65 \text{ ft-kip}$$

The moment distribution is determined as follows:

be	ba	bc
0.272	0.290	0.438
62.65	127.35	-336.00
+ 39.70	+ 42.30	+ 64.00
+102.35	+169.65	-272.00

$\xrightarrow{0.585}$ ab

ab
192.65
+ 24.75
+217.40

Thus M_{bc} = -272.0 ft-kip, M_{ba} = 169.65 ft-kip, M_{be} = 102.35 ft-kip, and M_{ab} = 217.40 ft-kip.

STRESSES ANALYSIS FOR SYMMETRIC LOADINGS CONDITION

For this loading condition, we can apply the conventional moment distribution method. A representative example is given as follows:

Example 4.—Determine the shrinkage stresses in the frame of example 3, if K_{bc} = 0.525 cu ft, K_{ba} = 0.35 cu ft, K_{be} = 0.49 cu ft, E = 420,000 kip per sq ft and shrinkage coefficient s = 0.0002. The solution is as follows:

The distribution factor and carry-over factor are determined to be

$$S_{bc} = 2\ 1.5 = 3.0 \qquad 0.268$$

$$S_{ba} = 4\ 1 \quad = 4.0 \qquad 0.357$$

$$S_{be} = 3\ 1.4 = \underline{4.2} \qquad \underline{0.375}$$

$$11.2 \qquad 1.000$$

and

$$Cs_{ba} = 0.5$$

The fixed end moments are

$$R_L = -\frac{Sh_1}{2\ L} = -\frac{0.0002\ 40}{2\ 28} = -0.000143$$

$$R_h = -\frac{S\ L}{h_3} = -\frac{0.0002\ 28}{50} = -0.000112$$

$$M_{F_{ba}} = M_{F_{ab}} = -6\ E\ K_{ba}\ R_L$$

$$= +6\ 420{,}000\ 0.35\ 0.000143 = +126.0 \text{ ft-kip}$$

$$M_{F_{be}} = +3\ 420{,}000\ 0.49\ 0.000112 = +\ 69.2 \text{ ft-kip}$$

$$M_{F_{bc}} = 0$$

The moment distribution is as follows:

be	ba	bc
0.375	0.357	0.268
+ 69.2	+126.0	
- 73.2	- 69.7	-52.3
- 4.0	+ 56.3	-52.3

0.5

ab
+126.0
- 34.85
+ 91.15

Thus M_{bc} = +52.3 ft-kip, M_{ba} = +56.3 ft-kip, M_{be} = -4.0 ft-kip, and M_{ab} = +91.15 ft-kip.

INFLUENCE LINE ANALYSIS

For a symmetric structure, any loadings may be considered as the combination of the symmetric and anti-symmetric ones. As shown in Fig. 16, loading (a) is equal to the sum of loadings (b), (c), (d), and (e). If only end moments are required, then (e) term vanishes, because this loading can not cause any moment at member ends of this structure. The values of M'_F, M''_F and P_m shown in Fig. 16 can be obtained as follows:

$$M_1 = k(1-k)^2\, h_1, \qquad M_2 = -k^2(1-k)h_1 \quad \ldots\ldots\ldots\ldots (15)$$

$$P_1 = 1-k + \frac{M_1+M_2}{h_1}, \qquad P_2 = k - \frac{M_1+M_2}{h_1} \quad \ldots\ldots\ldots\ldots (16)$$

$$M'_F = \frac{M_1+M_2}{2} = \frac{k(1-k)\ (1-2k)}{2}\, h_1 = C_4\, h_1 \quad \ldots\ldots\ldots\ldots (17)$$

$$M''_F = \frac{M_1-M_2}{2} = \frac{k(1-k)}{2}\, h_1 = C_3\, h_1 \quad \ldots\ldots\ldots\ldots (18)$$

$$P_m = \frac{P_1-P_2}{2} = \frac{1}{2} - k + k(1-k)\ (1-2\ k) = C_5 \quad \ldots\ldots\ldots\ldots (19)$$

These values are for the conditions when the unit load is applied on the central span of the structure. If the unit load is applied on the side span, loading (a) shown in Fig. 17 is equal to the sum of loadings (b), (c), and (d). In addition,

$$M_F = \frac{M}{2} = -\frac{1}{4}\, k\left(1-k^2\right)h_3 = -C_1h_3 \quad \ldots\ldots\ldots\ldots (20)$$

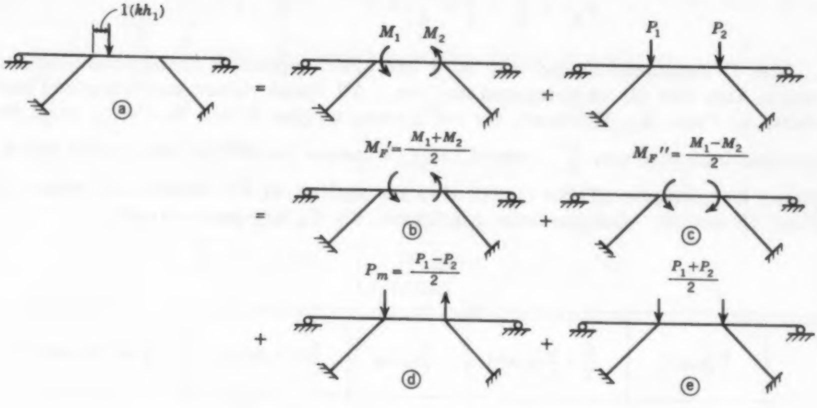


FIG. 16

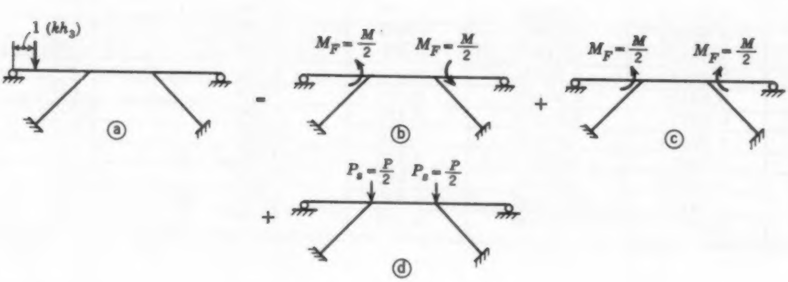
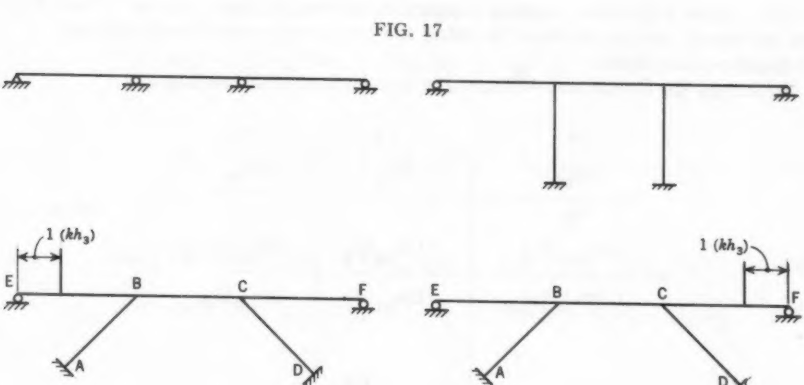


FIG. 17



FIGS. 18, 19, 20 AND 21

$$P_s = \frac{P}{2} = \frac{k}{2} + \frac{k}{4}\left(1-k^2\right) = C_2 \quad \ldots\ldots\ldots\ldots\ldots \quad (21)$$

The C-constants depend only on k and are independent of the span lengths, hence, they can all be computed earlier. All these values are computed and shown in Table 2. However, for the frames of type A and B, C_1 C_2 must be changed into zero and $\frac{k}{2}$, respectively, whereas the others remain the same. These C-constants (Table 2) can also be applied to the structures shown in Figs. 18 and 19. But for these conditions, C_2 C_5 are unnecessary.

TABLE 2

	$\frac{k}{4}\left(1-k^2\right)$	$\frac{k}{2}+\frac{k}{4}\left(1-k^2\right)$	$\frac{k}{2}(1-k)$	$\frac{k}{2}(1-k)(1-2k)$	$\frac{1}{2}-k+k(1-k)(1-2k)$
k	C_1	C_2	C_3	C_4	C_5
	load on	side span		load on	central span
0.0	0.00000	0.00000	0.000	0.000	0.500
0.1	0.02475	0.07475	0.045	0.036	0.472
0.2	0.04800	0.14800	0.080	0.048	0.396
0.3	0.06825	0.21825	0.105	0.042	0.284
0.4	0.08400	0.28400	0.120	0.024	0.148
0.5	0.09375	0.34375	0.125	0.000	0.000
0.6	0.09600	0.39600	0.120	-0.024	-0.148
0.7	0.08925	0.43925	0.105	-0.042	-0.284
0.8	0.07200	0.47200	0.080	-0.048	-0.396
0.9	0.04275	0.49200	0.045	-0.036	-0.472
1.0	0.00000	0.50000	0.000	0.000	-0.500

For those individual loading conditions shown in Figs. 16 and 17 the final end moments can be obtained by using the previously mentioned method.

Load on Side Span.—

Loadings (b) Condition.—These are anti-symmetric loadings.

be	ba	bc
Da_{be}	Da_{ba}	Da_{bc}
M_F		
$-Da_{be}\,M_F$	$-Da_{ba}M_F$	$-Da_{bc}M_F$
$\left(1-Da_{be}\right)M_F$	$-Da_{ba}M_F$	$-Da_{bc}M_F$

$$\xleftarrow{Ca_{ba}}$$

ab
$-Ca_{ba}Da_{ba}M_F$

Thus $M'_{bc} = -Da_{bc}\ M_F$, $M'_{ba} = -Da_{ba}\ M_F$, $M'_{be} = \left(1-Da_{be}\right)M_F$, and $M'_{ab} = -Ca_{ba}\ Da_{ba}\ M_F$.

Loadings (c) Condition.—These are symmetric loadings.

be	ba	bc
Ds_{be}	Ds_{ba}	Ds_{bc}
$+M_F$		
$-Ds_{be}M_F$	$-Ds_{ba}M_F$	$-Ds_{bc}M_F$
$\left(1-Ds_{be}\right)M_F$	$-Ds_{ba}M_F$	$-Ds_{bc}M_F$

$\xleftarrow{Ca_{ba}}$

ab
$-Cs_{ba}Ds_{ba}M_F$

Thus $M''_{bc} = -Ds_{bc}\ M_F$, $M''_{ba} = -Ds_{ba}\ M_F$, $M_{be} = \left(1-Ds_{be}\right)\ M_F$, and $M''_{ab} = -Cs_{ba}\ Ds_{ba}\ M_F$.

Loadings (d) Condition.—These are also anti-symmetric loadings. For this condition we may first assume $P_s = 10$, then find the end moments by the previous method (see Table 1 and example 5). Now, assume the final end moments are as follows:

$$\left.\begin{aligned} M'''_{bc} &= J_{bc}P_s, & M'''_{ba} &= J_{ba}P_s \\ M'''_{be} &= J_{be}P_s, & M'''_{ab} &= J_{ab}P_s \end{aligned}\right\} \qquad (22)$$

Since (a) = (b) + (c) + (d),

$$M_{bc}\ \text{(influence line)} = M'_{bc} + M''_{bc} + M'''_{bc} \qquad (23a)$$

$$M_{bc} = -Da_{be}M_F - Ds_{bc}M_F + J_{bc}P_s \qquad (23b)$$

and

$$M_{bc} = Da_{bc}h_3C_1 + Ds_{bc}h_3C_1 + J_{bc}C_2 \qquad (23c)$$

Similarly,

$$M_{ba}\ \text{(influence line)} = Da_{ba}h_3C_1 + Ds_{ba}h_3C_1 + J_{ba}C_2 \qquad (24)$$

$$M_{be}\ \text{(influence line)} = \left(Da_{be}-1\right)h_3C_1 + \left(Ds_{be}-1\right)h_3C_1 + J_{be}C_2 \qquad (25)$$

and

$$M_{ab}\ \text{(influence line)} = Ca_{ba}Da_{ba}h_3C_1 + Cs_{ba}Ds_{ba}h_3C_1 + J_{ab}C_2 \qquad (26)$$

Load on the Central Span.—

$$M_{bc}\ \text{(influence line)} = \left(1-Da_{bc}\right)h_1C_4 + \left(1-Ds_{bc}\right)h_1C_3 + J_{bc}C_5 \qquad (27)$$

$$M_{ba} \text{ (influence line)} = -Da_{be}h_1C_4 - Ds_{be}h_1C_3 + J_{be}C_5 \quad \ldots\ldots\ (28)$$

$$M_{be} \text{ (influence line)} = -Da_{be}h_1C_4 - Ds_{be}h_1C_3 + J_{be}C_5 \quad \ldots\ldots\ (29)$$

and

$$M_{ab} \text{ (influence line)} = -Ca_{ba}Da_{ba}h_1C_4 - Cs_{ba}Ds_{ba}h_1C_3 + J_{ab}C_5 \ldots\ (30)$$

Example 5.—Draw the end moment influence lines of the structure shown in example 3. The solution is as follows:

Step 1. Determine the factors (refer to example 3, 4)

$n_1 = 1.5$ $\quad n_2 = 1.4$ $\quad \alpha = 1.4$ $\quad \beta = 1.12$ $\quad A = 5.159$

$Da_{bc} = 0.438$ $\quad Da_{ba} = 0.290$ $\quad Da_{be} = 0.272$ $\quad Ca_{ba} = 0.585$

$Ds_{bc} = 0.268$ $\quad Ds_{ba} = 0.357$ $\quad Ds_{be} = 0.375$ $\quad Cs_{ba} = 0.500$

J-values can be obtained as follows:

Assume $P = 10$

refer to Fig. 10 and Table 1

$$M_{F_{ba}} = M_{F_{ab}} = \frac{10\ (96-40)}{2\ \ 5.159} = +54.2 \text{ ft-kip}$$

$$M_{F_{bc}} = -1.5\ \ 1.4\ \ 54.2 = -113.9$$

$$M_{F_{bc}} = \frac{1}{4}\ 1.4\ \ 1.12\ \ 54.2 = +21.25$$

The moment distribution then is

be	ba	bc
0.272	0.290	0.438
+21.25	+54.20	-113.90
+10.46	+11.15	+ 16.84
+31.71	+65.35	- 97.06

0.585

ab
+54.20
+ 6.52
+60.72

Thus $J_{bc} = -9.706$; $J_{ba} = 6.535$; $J_{be} = 3.171$, and $J_{ab} = 6.072$.

Step 2. Compute the left halves of the influence lines. For example, when the unit load is applied on the left side span, we obtain from Eq. 23c

$$M_{bc} = 21.9\ C_1 + 13.4\ C_1 - 9.706\ C_2$$

Substituting C_1 C_2 (Table 2) in the above equation, we can obtain the ordinates of M_{bc} influence line over the left side span. The detailed computations are shown in Table 3.

TABLE 3.—M_{bc} AND M_{ba}

k	$21.9C_1 + 13.4C_1 - 9.706C_2 = M_{hc}$	$14.5C_1 + 17.85C_1 + 6.535C_2 = M_{ba}$
	Load on the left side span	
0.1	0.5420 + 0.3318 - 0.7251 = 0.1487	0.3588 + 0.4420 + 0.4885 = 1.2893
0.2	1.0515 + 0.6438 - 1.4375 = 0.2578	0.6955 + 0.8565 + 0.9675 = 2.5195
0.3	1.4952 + 0.9147 - 2.1200 = 0.2899	0.9900 + 1.2198 + 1.4292 = 3.6390
0.4	1.8405 + 1.1245 - 2.7580 = 0.2070	1.2192 + 1.5000 + 1.8575 = 4.5767
0.5	2.0580 + 1.2575 - 3.3350 = -0.0195	1.3600 + 1.6735 + 2.2450 = 5.2785
0.6	2.1020 + 1.2870 - 3.8415 = -0.4520	1.3910 + 1.7133 + 2.5880 = 5.6923
0.7	1.9560 + 1.1951 - 4.2610 = -1.1099	1.2938 + 1.5925 + 2.8690 = 5.7553
0.8	1.5785 + 0.9649 - 4.5820 = -2.0386	1.0433 + 1.2850 + 3.0850 = 5.4133
0.9	0.9360 + 0.5732 - 4.7750 = -3.2658	0.6200 + 0.7638 + 3.2185 = 4.6023
	Load on the right side span	
0.1	-0.5420 + 0.3318 + 0.7251 = 0.5149	-0.3588 + 0.4420 - 0.4885 = -0.4035
0.2	-1.0515 + 0.6438 + 1.4375 = 1.0298	-0.6955 + 0.8565 - 0.9675 = -0.8065
0.3	-1.4952 + 0.9147 + 2.1200 = 1.5395	-0.9900 + 1.2198 - 1.4292 = -1.1994
0.4	-1.8405 + 1.1245 + 2.7580 = 2.0432	-1.2192 + 1.5000 - 1.8575 = -1.5767
0.5	-2.0580 + 1.2575 + 3.3350 = 2.5345	-1.3600 + 1.6735 - 2.2450 = -1.9325
0.6	-2.1020 + 1.2870 + 3.8415 = 3.0265	-1.3910 + 1.7133 - 2.5880 = -2.2657
0.7	-1.9560 + 1.1951 + 4.2610 = 3.5001	-1.2938 + 1.5925 - 2.8690 = -2.5703
0.8	-1.5785 + 0.9649 + 4.5820 = 3.9684	-1.0433 + 1.2850 - 3.0850 = -2.8433
0.9	-0.9360 + 0.5730 + 4.7750 = 4.4122	-0.6200 + 0.7638 - 3.2185 = -3.0750

All the values were computed by slide rule.

When the unit load is on the left half of the central span, the influence lines are similarly computed (see Table 5).

Step 3. Compute the right halves of the influence lines. This computation can be simplified by utilizing the relationship

$$M_{bc}\ (\text{Fig. 21}) = -M_{cb}\ (\text{Fig. 20}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (31a)$$

$$= -\left(Da_{bc}\ h_3 C_1 - Ds_{bc}\ h_3\ \ C_1 + J_{bc}\ C_2\right) \ldots\ldots \quad (31b)$$

$$= -\ Da_{bc}\ h_3\ C_1 - Ds_{bc}\ h_3\ C_1 - J_{bc}\ C_2 \ \ldots\ldots \quad (31c)$$

Comparing this with Eq. 23c, we can see that the corresponding terms of these equations are equal to each other except that some are opposite in signs. Hence, we can obtain the influence line ordinates over the right half of the

TABLE 4.—M_{be} AND M_{ab}

Load on the left side span

k	$-36.4C_1 - 31.25C_1 + 3.171C_2 = M_{be}$	$8.485C_1 + 8.925C_1 + 6.072C_2 = M_{ab}$
0.1	-0.9010 - 0.7740 + 0.2370 = -1.4328	0.2100 + 0.2220 + 0.4540 = 0.8860
0.2	-1.7475 - 1.5000 + 0.4693 = -2.7782	0.4073 + 0.4309 + 0.8990 = 1.7372
0.3	-2.4840 - 2.1330 + 0.6930 = -3.9240	0.5798 + 0.6135 + 1.3275 = 2.5208
0.4	-3.0595 - 2.6245 + 0.9010 = -4.7830	0.7133 + 0.7545 + 1.7240 = 3.1918
0.5	-3.4150 - 2.9285 + 1.0900 = -5.2535	0.7955 + 0.8415 + 2.0850 = 3.7220
0.6	-3.4965 - 2.9995 + 1.2550 = -5.2410	0.8149 + 0.8615 + 2.4035 = 4.0799
0.7	-3.2475 - 2.7890 + 1.3915 = -4.6450	0.7575 + 0.8009 + 2.6660 = 4.2244
0.8	-2.6208 - 2.2450 + 1.4975 = -3.3683	0.6110 + 0.6455 + 2.8650 = 4.1215
0.9	-1.5555 - 1.3520 + 1.5600 = -1.3475	0.3625 + 0.3837 + 2.9850 = 3.7312
	Load on the right side span	
0.1	0.9010 - 0.7740 - 0.2370 = -0.1100	-0.2100 + 0.2220 - 0.4540 = -0.4420
0.2	1.7475 - 1.5000 - 0.4693 = -0.2218	-0.4073 + 0.4309 - 0.8990 = -0.8754
0.3	2.4840 - 2.1330 - 0.6930 = -0.3420	-0.5798 + 0.6135 - 1.3275 = -1.2938
0.4	3.0595 - 2.6245 - 0.9010 = -0.4660	-0.7133 + 0.7545 - 1.7240 = -1.6828
0.5	3.4150 - 2.9285 - 1.0900 = -0.6035	-0.7955 + 0.8415 - 2.0850 = -2.0391
0.6	3.4965 - 2.9995 - 1.2550 = -0.7580	-0.8149 + 0.8615 - 2.4035 = -2.3569
0.7	3.2475 - 2.7890 - 1.3915 = -0.9330	-0.7575 + 0.8009 - 2.6660 = -2.6226
0.8	2.6208 - 2.2450 - 1.4975 = -1.1217	-0.6110 + 0.6445 - 2.8650 = -2.8305
0.9	1.5555 - 1.3520 - 1.560 = -1.3565	-0.3625 + 0.3837 - 2.9850 = -2.9638

All the values were computed by slide rule.

TABLE 5.—M_{bc} AND M_{ba}

k	$22.48C_4 + 29.29C_3 - 9.706C_5 = M_{bc}$	$-11.6C_4 - 14.29C_3 + 6.535C_5 = M_{ba}$
0.0	- 4.8530 = -4.8530	3.2675 = 3.2675
0.1	0.8100 + 1.3190 - 4.5810 = -2.4500	-0.4175 - 0.6432 + 3.0820 = 2.0213
0.2	1.0800 + 2.3420 - 3.8420 = -0.4200	-0.5562 - 1.1420 + 2.5880 = 0.8898
0.3	0.9450 + 3.0770 - 2.7580 = 1.2640	-0.4870 - 1.5010 + 1.8575 = 0.1305
0.4	0.5400 + 3.5180 - 1.4380 = 2.6200	-0.2782 - 1.7140 + 0.9668 = -1.0254
0.5	3.6620 = 3.6620	- 1.7850 = -1.7850
0.6	-0.5400 + 3.5180 + 1.4380 = 4.4160	0.2782 - 1.7140 - 0.9668 = -2.4046
0.7	-0.9450 + 3.0770 + 2.7580 = 4.8900	0.4870 - 1.5010 - 1.8575 = -2.8715
0.8	-0.0800 + 2.3420 + 3.8420 = 5.1040	0.5562 - 1.1420 - 2.5880 = -3.1738
0.9	-0.8100 + 1.3190 + 4.5810 = 5.0900	0.4175 - 0.6432 - 3.0820 = -3.3077
1.0	4.8530 = 4.8530	- 3.2675 = -3.2675

TABLE 6.—M_{be} AND M_{ab}

k	$-10.88C_4 - 15.05C_3 + 3.171C_5 = M_{be}$	$-6.79C_4 - 7.14C_3 + 6.072C_5 = M_{ab}$
0.0	1.5855 = 1.5855	3.0360 = 3.0360
0.1	-0.3917 - 0.6775 + 1.4975 = 0.3280	-0.2442 - 0.3213 + 2.8620 = 2.2965
0.2	-0.5220 - 1.2045 + 1.2570 = -0.4695	-0.3260 - 0.5710 + 2.4050 = 1.5080
0.3	-0.4568 - 1.5810 + 0.9010 = -1.1368	-0.2853 - 0.7500 + 1.7233 = 0.6880
0.4	-0.2611 - 1.8080 + 0.4695 = -1.5996	-0.1630 - 0.8570 + 0.8995 = -0.1205
0.5	- 1.8810 = -1.8810	- 0.8938 = -0.8938
0.6	0.2611 - 1.8080 - 0.4695 = -2.0164	0.1630 - 0.8570 - 0.8995 = -1.5935
0.7	0.4568 - 1.5810 - 0.9010 = -2.0252	0.2853 - 0.7500 - 1.7233 = -2.1880
0.8	0.5220 - 1.2045 - 1.2570 = -1.9295	0.3260 - 0.5710 - 2.4050 = -2.6500
0.9	0.3917 - 0.6775 - 1.4975 = -1.7833	0.2442 - 0.3213 - 2.8620 = -2.9391
1.0	- 1.5855 = -1.5855	- 3.0360 = -3.0360

All the values were computed by slide rule.

structure by summing up the corresponding terms of the left half span after making some sign modifications. (See Tables 3 and 5.)

CONCLUSION

The advantages of this method are its directness and simplicity. Although the computations of stiffness and carry-over factor are rather complicated,

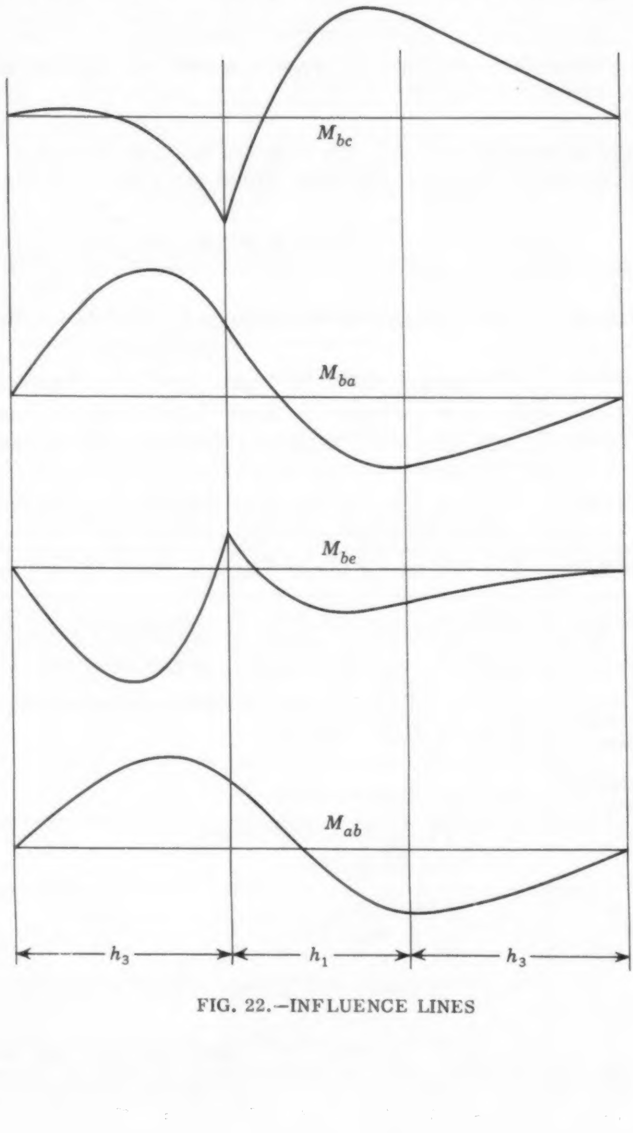


FIG. 22.—INFLUENCE LINES

except for a given bridge, these values are only computed once and then we can apply that to the different analyses.

ACKNOWLEDGMENT

Thanks are due to Mr. K. K. Hu for his valuable assistance in the preparation of this paper.

APPENDIX I.—FRAMES OF VARYING SECTION MEMBERS

For varying section members, the formulas for anti-symmetric loadings condition can also be derived. All these values are shown in Tables 7 and 8.

TABLE 7.—TYPE D VARYING SECTION MEMBER

S_{ba}	absolute	$\frac{1}{A}(1-C_{BA}C_{AB})\left[n_1\alpha(\alpha+1)(1+C_{BC})-\frac{1}{4}n_2\beta(2-\beta)(1-C_{BE}C_{EB})\right]EK_{BA}$
	relative	$(1-C_{BA}C_{AB})\left[n_1\alpha(\alpha+1)(1+C_{BC})-\frac{1}{4}n_2\beta(2-\beta)(1-C_{BE}C_{EB})\right]$
S_{bc}	absolute	$\frac{1}{A}(1+C_{BC})\left[(\alpha+1)(1-C_{BA}C_{AB})+\frac{1}{4}n_2\beta(2\alpha-\beta)(1-C_{BE}C_{EB})\right]EK_{BC}$
	relative	$n_1(1+C_{BC})\left[(\alpha+1)(1-C_{BA}C_{AB})+\frac{1}{4}n_2\beta(2\alpha-\beta)(1-C_{BE}C_{EB})\right]$
S_{be}	absolute	$\frac{1}{A}(1-C_{BE}C_{EB})\left[\left(1-\frac{1}{2}\beta\right)(1-C_{BA}C_{AB})+n_1\alpha\left(\alpha+\frac{1}{2}\beta\right)(1+C_{BC})\right]EK_{BE}$
	relative	$n_2(1-C_{BE}C_{EB})\left[\left(1-\frac{1}{2}\beta\right)(1-C_{BA}C_{AB})+n_1\alpha\left(\alpha+\frac{1}{2}\beta\right)(1+C_{BC})\right]$
Ca_{ba}		0
$M_{F_{ba}}$ or $M_{F_{ab}}$	vertical loads	$\frac{1}{2A}(1-C_{BA}C_{AB})\,P(h_2-h_1)$
	horizontal load	$\frac{1}{2A}(1-C_{BA}C_{AB})\,PL$
$M_{F_{bc}}$		$-n_1\alpha\,\frac{1+C_{BC}}{1-C_{BA}C_{AB}}\,M_{F_{ba}}$
$M_{F_{be}}$		$\frac{1}{2}n_2\beta\,\frac{1-C_{BE}C_{EB}}{1-C_{BA}C_{AB}}\,M_{F_{ba}}$
Remarks		$A=(1-C_{BA}C_{AB})+n_1\alpha^2(1+C_{BC})+\frac{1}{4}n_2\beta^2(1-C_{BE}C_{EB})$ $R_L=\frac{1}{A}\left[(1-C_{BA}C_{AB})+n_1\alpha(1+C_{BC})+\frac{1}{4}n_2\beta(1-C_{BE}C_{EB})\right]\theta$

TABLE 8.—TYPE C VARYING SECTION MEMBER

S_{ba}	absolute	$\frac{1}{A}\left\{n_1\alpha(1+C_{BC})[\alpha+(1+C_{BA})]+\frac{1}{4}n_2\beta(1-C_{BE}C_{EB})[\beta-2(1+C_{BA})]+n'(1-C_{BA}C_{AB})\right\}EK_{BA}$
	relative	$n_1\alpha(1+C_{BC})[\alpha+(1+C_{BA})]+\frac{1}{4}n_2\beta(1-C_{BE}C_{EB})[\beta-2(1+C_{BA})]+n'(1-C_{BA}C_{AB})$
S_{bc}	absolute	$\frac{1}{A}(1+C_{BC})\left[\alpha(1+n'C_{AB})+\frac{1}{4}n_2\beta(1-C_{BE}C_{EB})(2\alpha+\beta)+n'(1+C_{AB})+(1+C_{BA})\right]EK_{BC}$
	relative	$n_1(1+C_{BC})\left[\alpha(1+n'C_{AB})+\frac{1}{4}n_2\beta(1-C_{BE}C_{EB})(2\alpha+\beta)+n'(1+C_{AB})+(1+C_{BA})\right]$
S_{be}	absolute	$\frac{1}{A}(1-C_{BE}C_{EB})\left[n_1\alpha(1+C_{BC})\left(\alpha+\frac{1}{2}\beta\right)-\frac{1}{2}\beta(1+n'C_{AB})+n(1+C_{BA})+(1+C_{AB})\right]EK_{BE}$
	relative	$n_2(1-C_{BE}C_{EB})\left[n_1\alpha(1+C_{BC})\left(\alpha+\frac{1}{2}\beta\right)-\frac{1}{2}\beta(1+n'C_{AB})+n'(1+C_{BA})+(1+C_{AB})\right]$
Ca_{ba}		$\frac{1}{Sr_{ba}}n'\left\{n_1\alpha(1+C_{BC})[C_{AB}(\alpha+1)+1]-\frac{1}{4}n_2\beta(1-C_{BE}C_{EB})[C_{AB}(2-\beta)+2]-(1-C_{BA}C_{AB})\right\}$
$M_{F_{ba}}$ or $M_{F_{ab}}$	vertical loads	$\frac{1}{2A}(1+C_{BA})\ P(h_2-h_2)$
	horizontal load	$\frac{1}{2A}(1+C_{BA})\ PL$
$M_{F_{bc}}$		$-n_1\alpha\frac{1+C_{BC}}{1+C_{BA}}M_{F_{ba}}$
$M_{F_{be}}$		$\frac{1}{2}n_2\beta\frac{1-C_{BE}C_{EB}}{1+C_{BA}}M_{F_{ba}}$
Remarks		$A=n_1\alpha^2(1+C_{BC})+\frac{1}{4}n_2\beta^2(1-C_{BE}C_{EB})+(1+C_{BA})+n'(1+C_{AB})$ $R_L=\frac{1}{A}[1+n'C_{AB}-n_1\alpha(1+C_{BC})+\frac{1}{2}n_2\beta(1-C_{BE}C_{EB})]\theta$

The notations are the same as previously used except that K is the moment that, when applied at simply supported end of a beam, will cause unit rotation there when the far end is fixed against rotation (no sidesway), C refers to the carry-over factor (no sidesway), and $n' = \frac{K_{AB}}{K_{BA}}$.

APPENDIX II.—NOTATION

$$\alpha = \frac{h_2-h_1}{h_1};\quad \beta = \frac{h_2-h_1}{h_3};\quad n_1 = \frac{K_{h_1}}{K_L};\quad n_2 = \frac{K_{h_3}}{K_L};$$

$$K = \frac{I}{L};$$

I = moment of inertia;

L = length of member;

E = modulus of elasticity;

Sa = absolute stiffness;

Sr = relative stiffness;

Da = distribution factor for anti-symmetric loadings condition;

Ds = distribution factor for symmetric loadings condition;

Ca = carry-over factor for anti-symmetric loadings condition;

Cs = carry-over factor for symmetric loadings condition; and

M_F = fixed end moment.

The subscript L denotes the inclined member and h the horizontal member.

Sign Conventions.—Moments, slopes . . . are considered as positive if they are counterclockwise.

Journal of the
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

MOVEMENTS OF A CABLE DUE TO CHANGES IN LOADING

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SYNOPSIS

A numerical method is presented for the determination of displacements along a suspended cable resulting from change in load. The effect of elastic deformations is included. All computations in the numerical examples were made with the aid of a desk calculator. The procedure, however, is particularly suitable for programming for an electronic digital computer.

INTRODUCTION

The determination of displacements of a cable due to changes in loading is important, not only for computing cable stresses but also for planning the erection of suspended structures. Although the literature on cables and suspended structures is extensive, no direct solution seems to be available for determining the displacements of a cable under a general system of loading where the movements are caused by changes in magnitude or position of the loads. Solutions for particular loading arrangements have been presented by many inves-

Note.—Discussion open until May 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 12, December, 1960.

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tigators.[3,4,5,6,7,8,9,10,11] However, since superposition is not permissible, such individual solutions cannot be combined to obtain solutions for more general cases of loading.

The procedure presented herein is applicable to any loading pattern and yields results through a series of successive approximations.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

MOVEMENTS OF THE CABLE

For most structures the displacements resulting from loading are so small that their effect on the primary bending moments, shears, and axial forces may be neglected. The principle of superposition is applicable. On the other hand, cable displacements that result from change in load are, in general, so large that they must be considered in the primary stress analysis. Hence the principle of superposition cannot be applied.

Fig. 1 shows a cable supporting loads W_1 and W_2. The initial position is the equilibrium (or string) polygon formed by the applied loads. For the present the dead load of the cable itself is neglected. The polygon $C_0C_1C_2C_3C_0$ is the moment diagram to some scale of an analogous simply-supported beam loaded with W_1 and W_2. Thus, for a particular loading, if the location of one point on the cable in addition to the support points is known, the cable curve is completely defined. The horizontal component of the cable tension is equal to the simple-beam bending moment at any point on the cable, M_3, divided by the corresponding vertical distance, e, between that point and the chord C_0C_3. That is,

$$H_w = \frac{M_s}{e} \qquad (1)$$

The subscript w, as used herein, refers to the cable in its original position, whereas the subscript p refers to the cable in its displaced position. The slope of any cable segment is equal to the ratio of the vertical component to the horizontal component of cable tension. That is,

$$\tan\theta = \frac{V_w}{H_w} \qquad (2)$$

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3 "A Simple Method of Computing Deflections of a Cable Span Carrying Multiple Loads Evenly Spaced," by F. C. Carstarphen, Transactions, ASCE, Vol. 83, 1919, pp. 1383-1408.

4 "Seilschwebebahen," by E. Czitary, Springer Verlag, Vienna, 1951.

5 "Unbraced Cables," by Jacob Feld, Journal, Franklin Inst., Vol. 209, No. 1, January, 1930, pp. 83-108.

6 "Modern Framed Structures," by J. B. Johnson, C. Bryan, and F. E. Turneaure, Part 2, Ninth Ed., John Wiley and Sons, New York, 1911.

7 "Deflection of Cable due to a Single Point Load," by E. Markland, Philosophical Magazine, Vol. 42, No. 33, September, 1951.

8 "The Theory of Suspension Bridges," by A. Pugsley, Edward Arnold, Ltd., London.

9 "Uber das schwere Seil mit einer Einzelkraft," by F. Schleicher, Bauingenieur, Vol. 12, No. 46, November 13, 1931.

10 "Suspension Bridges," by D. B. Steinman, Second Ed., John Wiley and Sons, New York, 1929.

11 "Theory of Suspension Bridges," by S. P. Timoshenko, Journal, Franklin Inst., Vol. 235, No. 3, 1943.

The vertical component of cable segment tension, V_w, is equal to the shear in the corresponding simply-supported beam, corrected by the quantity $\frac{H_w h}{L}$, in which L is the span of the cable and h is the difference in elevation between cable ends.

When loads P_1 and P_2 are added to the initial loads, the cable is displaced as shown in Fig. 1. The displacements of points C_1 and C_2 are the result of the change in shape of the equilibrium polygon plus the elastic change in length of the cable. The magnitude of these displacements depends on the ratio of initial load to live load, the dissymmetry of the live load, the initial cable sag, and the elastic properties of the cable.

Since the horizontal component of cable tension, H_p, is a function of the new cable shape, which is in turn a function of H_p, the displaced position of the cable

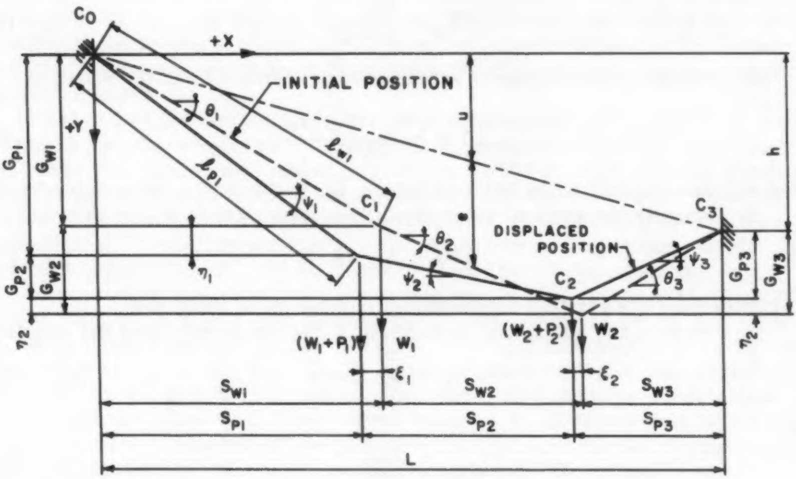


FIG. 1.—CABLE DISPLACED DUE TO CHANGE IN LOAD

in Fig. 1 cannot be determined directly from the simple-beam bending moment diagram. A further complicating factor results from the horizontal displacements of the cable.

COMPUTATIONAL PROCEDURE

In the proposed method a value of H_p is assumed. On the basis of this assumption corresponding values of shears, V_p, are determined by successive adjustment such that they satisfy the condition of equilibrium about the support points. However, unless the assumed value of H_p is the correct one, the boundary conditions will not be satisfied, that is, displacements η and ξ of one end of the cable with respect to the other will be indicated. A new value of H_p is then assumed and the procedure repeated. On the basis of two or more such cycles, the correct value of H_p is obtained from a plot of end displacements versus trial values of H_p. With the correct value of H_p, the procedure of suc-

cessive adjustments is then used to compute the vertical and horizontal displacements of the load points. The relative displacements of the cable ends will equal zero (or practically so).

The procedure may be summarized as follows:

1. Assume a trial value of H_p.

2. Assume a value of cable shear, V_p, for the left end cable segment and compute the corresponding cable shears in the other segments.

3. Determine the value of $\tan \psi$ for each segment from the equation

$$\tan \psi = \frac{V_p}{H_p} \qquad (3)$$

4. Determine the tension in each segment from the equation

$$T_p = \frac{H_p}{\cos \psi} \qquad (4)$$

5. Determine the change in length of each segment from the equation

$$\Delta l = \frac{(T_p - T_w)\, l_w}{A\, E} \qquad (5)$$

and add this quantity to the original length, l_w, to determine the new length, l_p.

6. Compute the vertical projection of each segment from the equation

$$G_p = l_p \sin \psi \qquad (6)$$

and determine the vertical displacement η of the cable right end.

7. Compute the horizontal projection of each segment from the equation

$$S_p = l_p \cos \psi \qquad (7)$$

or

$$S_p = \frac{G_p}{\tan \psi} \qquad (8)$$

and determine the horizontal displacement ξ of the cable right end.

8. Compute the moment of the forces about the right end support based on the displaced position of the load points. If the moment is not zero, compute the shear correction necessary to make it zero. The shear correction is equal to the moment divided by the span.

9. Using the shear correction from step 8 as a guide, select new values of V_p and repeat steps 3 through 8 (using the same value of H_p) until the shear correction of step 8 becomes negligible. Determine the displacements of the cable end and plot them against the assumed value of H_p.

10. Repeat steps 1 through 9 for other trial values of H_p.

11. From the plot select a value of H_p for which the displacements η and ξ are zero, and repeat steps 2 through 9 to obtain the final data for the displaced cable.

EXAMPLE 1

In Table 1 a 3/4-in. diameter wire rope having a cross-sectional area of 0.227 sq in. and a modulus of elasticity of 12,000,000 psi supports loads W of

4 kips at each of the load points C_1 and C_2. It is desired to determine the movements of the cable when a load P of 3 kips is added at C_2. In this example the weight of the cable is neglected. The given values of G_w, S_w, l_w, W, V_w, H_w, T_w, P, and W + P are listed on lines 1 through 9. Downward forces are considered negative.

A trial value of H_p equal to 10.9 kips is assumed. The tabulation from lines 10 through 18 follows as outlined in procedural steps 2 through 7. A positive sign for η_3 indicates a downward displacement of the cable end while a positive sign for ξ_3 indicates displacement to the left. A summation of moments is then made about an axis through the right support. It is convenient, particularly when many segments are involved, to proceed as shown on line 19. On that line are listed the products of V_p (from line 10) and the distances S_p (from line 18). Note that for the last segment ξ_3 is added to S_p. The sum of the values on line 19, +15.838 ft.-kips, is the sum of the moments about an axis through the support at C_3. The shear correction is -15.838/300, or -0.0528 kip. This is added to the values of V_p on line 10 to arrive at new values for V_p, listed on line 20. The tabulation for lines 20 through 29 follows in accordance with procedural step 9.

At the end of the second cycle, the displacements η_3 and ξ_3 shown on lines 27 and 28 are smaller than the corresponding displacements from the first cycle. Note that the moment about support C_3 (3.394 ft-kips) is still positive although of smaller magnitude. In successive approximations the shear correction diminishes, the sign remaining negative, until the fifth cycle where it is negligible. This suggests that if a shear correction larger than -0.0528 kip had been used on line 20, the convergence would have been speeded.

The procedure was repeated for assumed values of H_p equal to 11.00, 11.02, 11.025, and 11.10 kips. The corresponding displacements of η_3 and ξ_3 are shown in Table 2 and are plotted in Fig. 2. For each assumed value of H_p, the final value of V_p for segment C_o C_1 is also plotted in Fig. 2. As successive values of η_3, ξ_3, and V_p are plotted, they serve as a guide in the selection of new values of H_p and V_p. The correct value of H_p is determined as 11.0216 kips from the intersection of the plot of η_3 (or ξ_3) with the zero displacement axis. Theoretically, this is a common intersection. The probable value of V_p for segment C_o C_1 is determined from Fig. 2 as being equal to 4.9252 kips.

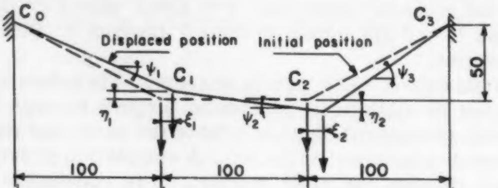
Using H_p equal to 11.0216 kips, and V_p equal to 4.9252 kips for segment C_o C_1, the procedure of successive adjustments was repeated as shown in Table 3. After the second cycle the displacements at support C_3 are practically zero, as shown on lines 18 and 19, and the moment is practically zero about support C_3, as shown in line 20. The correct displacements at the load points are shown on lines 21 and 22.

EXAMPLE 2

In Table 4, a 1 1/8-in. diameter locked coil strand spans 1,000 ft between supports at the same elevation. It has a weight of 3.16 lb per ft, a cross-sectional area of 0.85 sq in., and a modulus of elasticity equal to 19,000,000 psi. At design temperature the sag at midspan is 100 ft. The cable displacements, when an 8-kip live load is placed 400 ft from the left support, are to be determined.

To approximate the effect of the cable dead weight, the curved initial position of the cable was replaced by ten straight segments with the dead load concentrated at the intersections. The ordinates to the initial position of the cable,

TABLE 1



1	G_w		50.0000		0.0000		-50.0000		ft.
2	S_w		100.0000		100.0000		100.0000		ft.
3	l_w		111.8034		100.0000		111.8034		ft.
4	W	4.0		-4.0		-4.0		4.0	kip
5	V_w		4.0		0.0		-4.0		kip
6	H_w		8.0		8.0		8.0		kip
7	T_w		8.9443		8.0000		8.9443		kip
8	P	1.0		0.0		-3.0		2.0	kip
9	W + P	5.0		-4.0		-7.0		6.0	kip
	ASSUME H_p = 10.9 KIPS - FIRST APPROXIMATION TO DISPLACEMENTS								
10	V_p		5.0		1.0		-6.0		kip
11	tan Ψ		.4587156		.0917431		-.5504587		
12	sin Ψ		.4169418		.0913595		-.4822272		
13	cos Ψ		.9089331		.9958188		.8760461		
14	T_p		11.9921		10.9458		12.4423		kip
15	Δl		.1251		.1081		.1436		ft.
16	l_p		111.9285		100.1081		111.9470		ft.
17	G_p		46.668		9.146		-53.984		$\eta_3 = \Sigma G_p = 1.830$ ft.
18	S_p		101.736		99.690		98.071		$\xi_3 = L - \Sigma S_p = -.503$ ft.
19	V_pS_p		508.680		98.602		-591.444		$\Sigma = 15.838$ ft. kip
									shear corr. = $\frac{-15.838}{300}$ = -.0528 kip
	SECOND APPROXIMATION TO DISPLACEMENTS								
20	V_p		4.9472		.9472		-6.0528		
21	tan Ψ		.4538716		.0868991		-.5553028		
22	sin Ψ		.4132944		.0865703		-.4854740		
23	cos Ψ		.9105976		.9962163		.8742509		
24	T_p		11.9702		10.9414		12.4678		kip
25	Δl		.1242		.1080		.1446		ft.
26	l_p		111.9276		100.1080		111.9480		ft.
27	G_p		46.259		8.666		-54.348		η_3 = .577 ft.
28	S_p		101.921		99.729		97.871		ξ_3 = .479 ft.
29	V_pS_p		504.224		94.463		-595.293		Σ = 3.394 ft. kip

TABLE 1.—CONT'D

		C_0	C_1	C_2	C_3	
		THIRD APPROXIMATION TO DISPLACEMENTS				
30	V_p	4.9359	.9359	-6.0641	corr. = -.0113	kip
31	tan Ψ	.4528349	.0858624	-.5563394		
32	sin Ψ	.4125112	.0855476	-.4861662		
33	cos Ψ	.9109528	.9963337	.8738662		
34	T_p	11.9655	10.9401	12.4733		kip
35	Δl	.1240	.1079	.1448		ft.
36	l_p	111.9274	100.1079	111.9482		ft.
37	G_p	46.171	8.564	-54.425	η_3 = .310	ft.
38	S_p	101.961	99.741	97.828	ξ_3 = .470	ft.
39	V_pS_p	503.269	93.348	-596.089	Σ = .528	ft. kip
		FOURTH APPROXIMATION TO DISPLACEMENTS				
40	V_p	4.9341	.9341	-6.0659	corr. = -.0018	kip
41	tan Ψ	.4526697	.0856972	.5565046		
42	sin Ψ	.4123862	.0853842	.4862765		
43	cos Ψ	.9110093	.9963476	.8738050		
44	T_p	11.9648	10.9400	12.4742		kip
45	Δl	.1240	.1079	.1449		ft.
46	l_p	111.9274	100.1079	111.9483		ft.
47	G_p	46.157	8.548	-54.438	η_3 = .267	ft.
48	S_p	101.967	99.742	97.821	ξ_3 = .470	ft.
49	V_pS_p	503.115	93.169	-596.223	Σ = .061	ft. kip
		FIFTH APPROXIMATION TO DISPLACEMENTS				
50	V_p	4.9339	.9339	-6.0661	corr. = -.0002	kip
51	tan Ψ	.4526514	.0856789	.5565229		
52	sin Ψ	.4123724	.0853661	.4862934		
53	cos Ψ	.9110154	.9963492	.8738066		
54	T_p	11.9647	10.9399	12.4742		kip
55	Δl	.1240	.1079	.1449		ft.
56	l_p	111.9274	100.1079	111.9483		ft.
57	G_p	46.156	8.546	-54.440	η_3 = .262	ft.
58	S_p	101.968	99.742	97.821	ξ_3 = .469	ft.
59	V_pS_p	503.100	93.149	-596.237	Σ = .012	ft. kip
					corr. = -.0004 neglect	
		DISPLACEMENTS WITH ASSUMED VALUE OF H_p = 10.90 KIPS				
60	η	0.0	-3.844	4.702	.262	ft.
61	ξ	0.0	-1.968	-1.710	.469	ft.

TABLE 2

H_p kips (1)	η_1 (2)	ξ_1 (3)	η_2 (4)	ξ_2 (5)	η_3 (6)	ξ_3 (7)
10.900	-3.844	-1.968	4.702	-1.710	0.262	0.469
11.000	-4.247	-2.153	4.157	-1.912	0.046	0.082
11.020	-4.327	-2.190	4.050	-1.951	0.004	0.006
11.025	-4.347	-2.199	4.023	-1.961	-0.007	-0.013
11.100	-4.644	-2.333	3.624	-2.106	-0.161	-0.295

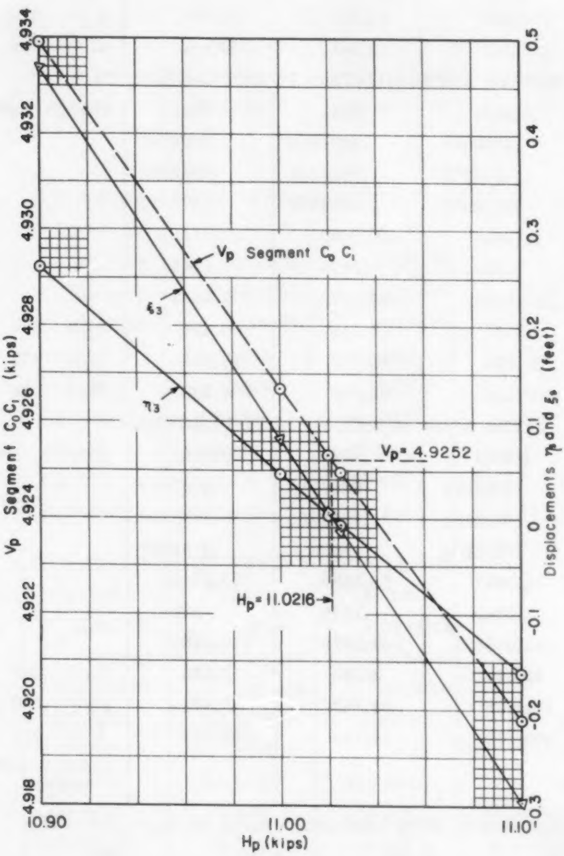


FIG. 2.—PLOT OF η_3, ξ_3, AND V_p VERSUS H_p FOR EXAMPLE 1

shown on line 1, were computed from the following equation of a catenary with the origin at C_5:

$$y = -\left[\frac{H_w}{w}\left(\cosh\frac{wx}{H_w} - 1\right)\right] \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

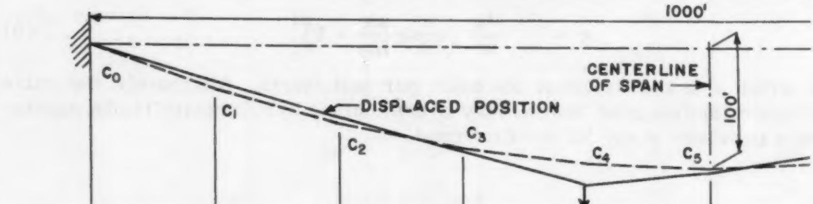
in which w is the weight of the cable per unit length. Alternately, the cable ordinates and segment lengths may be determined by successive trails, assuming a parabolic shape for the first trial.

TABLE 3

H_p = 11.0216	C_0	C_1	C_2	C_3	
	FIRST APPROXIMATION TO DISPLACEMENTS				
1 V_p	4.9252	.9252	-6.0748		kip
2 tan Ψ	.446879	.0839442	.5511722		
3 sin Ψ	.4079855	.0836501	.4827067		
4 cos Ψ	.9129890	.9964964	.8757820		
5 T_p	12.0720	11.0603	12.5849		kip
6 Δl	.1284	.1123	.1494		ft.
7 l_p	111.9318	100.1123	111.9528		ft.
8 G_p	45.6666	8.3744	-54.0404	η_3 = .0006	ft.
9 S_p	102.1925	99.7615	98.0462	ξ_3 = -.0002	ft.
10 V_pS_p	503.319	92.2993	-595.610	Σ = .008	ft. kip
	SECOND APPROXIMATION TO DISPLACEMENTS				
11 V_p	4.92517	.92517	-6.07483	corr. = -.00003	kip
12 tan Ψ	.4468652	.0839415	.5511749		
13 sin Ψ	.4079832	.0836474	.4827085		
14 cos Ψ	.9129894	.9964963	.8757809		
15 T_p	12.0720	11.0603	12.5849		kip
16 Δl	.1284	.1123	.1494		ft.
17 l_p	111.9318	100.1123	111.9528		ft.
18 G_p	45.6663	8.3741	-54.0406	η_3 = -.0002	ft.
19 S_p	102.1925	99.7615	98.0461	ξ_3 = -.0001	ft.
20 V_pS_p	503.315	92.296	-595.613	Σ = -.002	ft. kip
				corr. = .00001 neglect	
	DISPLACEMENTS WITH CORRECT VALUE OF H_p = 11.0216 KIPS				
21 η	0.0	-4.334	4.040	0.000	ft.
22 ξ	0.0	-2.193	-1.954	0.000	ft.

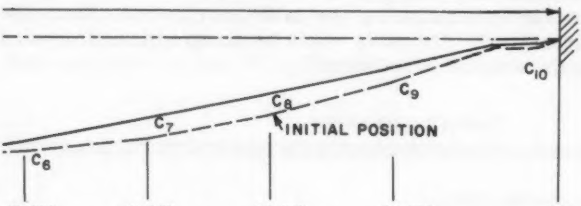
The vertical and horizontal projections of the cable segments in the initial position, listed on lines 2 and 3 respectively, were used to calculate the segment lengths shown on line 4. Concentrated dead loads are shown on line 5, and total loads are listed on line 10.

TABLE 4



1	e	0.0		36.300		64.300		84.175		96.050		100.000	
2	G_w		36.300		28.000		19.875		11.875		3.950		-3.950
3	S_w		100.000		100.000		100.000		100.000		100.000		100.000
4	l_w		106.3846		103.8460		101.9559		100.7026		100.0780		100.0780
5	W	1.452		-.332		-.325		-.320		-.317		-.316	
6	V_w		1.452		1.120		.795		.475		.158		-.158
7	H_w		4.000		4.000		4.000		4.000		4.000		4.000
8	T_w		4.255		4.154		4.078		4.028		4.003		4.003
9	P	4.800								-8.000			
10	W + P	6.252		-.332		-.325		-.320		-8.317		-.316	
	ASSUME H_p = 19.50 KIPS FIRST APPROXIMATION												
11	V_p		6.2520		5.9200		5.5950		5.2750		-3.0420		-3.3580
12	tan ψ		.3206154		.3035897		.2869231		.2705128		-.1560000		-.1722051
13	sin ψ		.3053073		.2904976		.2757952		.2611272		-.1541358		-.1697071
14	cos ψ		.9522540		.9568757		.9612164		.9653044		.9880500		.9854940
15	T_p		20.478		20.379		20.287		20.201		19.736		19.787
16	Δl		.1069		.1043		.1023		.1008		.0975		.0978
17	l_p		106.4915		103.9503		102.0582		100.8034		100.1755		100.1758
18	G_p		32.5126		30.1973		28.1472		26.3225		-15.4406		-17.0005
19	S_p		101.4070		99.4675		98.1000		97.3060		98.9784		98.7226
20	V_pS_p		633.997		588.848		548.870		513.289		-301.092		-331.510
	SECOND APPROXIMATION TO												
21	V_p		6.2950		5.9630		5.6380		5.3180		-2.9990		-3.3150
22	tan ψ		.3228205		.3057948		.2891282		.2727179		-.1537948		-.1700000
23	sin ψ		.3072095		.2924279		.2777518		.2631090		-.1520076		-.1675955
24	cos ψ		.9516419		.9562880		.9606527		.9647661		.9883793		.9858558
25	T_p		20.491		20.391		20.299		20.212		19.729		19.780
26	Δl		.1070		.1044		.1024		.1009		.0975		.0978
27	l_p		106.4915		103.9503		102.0582		100.8034		100.1755		100.1758
28	G_p		32.7152		30.3980		28.3468		26.5223		-15.2274		-16.7890
29	S_p		101.3418		99.4064		98.0425		97.2517		99.0114		98.7589
30	V_pS_p		637.947		592.760		552.764		517.185		-296.935		-327.386
	THIRD APPROXIMATION TO												
31	V_p		6.2945		5.9625		5.6375		5.3175		-2.9995		-3.3155
32	tan ψ		.3227949		.3057692		.2891026		.2726923		-.1538205		-.1700256
33	sin ψ		.3071875		.2924054		.2777291		.2630860		-.1520323		-.1676201
34	l_p		106.4915		103.9503		102.0582		100.8034		100.1755		100.1758
35	G_p		32.7129		30.3956		28.3445		26.5200		-15.2299		-16.7915
36	S_p		101.3425		99.4071		98.0431		97.2523		99.0109		98.7585
37	V_pS_p		637.900		592.715		552.718		517.139		-296.983		-327.434

TABLE 4. –*CONTINUED*



96.050		84.175		64.300		36.300		0.0		ft.	1
	-11.875		-19.875		-28.000		-36.300			ft.	2
	100.000		100.000		100.000		100.000			ft.	3
	100.7026		101.9559		103.8460		106.3846			ft.	4
-.317		-.320		-.325		-.332		1.452		kip	5
	-.475		-.795		-1.120		-1.452			kip	6
	4.000		4.000		4.000		4.000			kip	7
	4.028		4.078		4.154		4.255			kip	8
								3.200		kip	9
-.317		-.320		-.325		-.332		4.652		kip	10
TO DISPLACEMENTS											
	-3.6750		-3.9950		-4.3200		-4.6520			kip	11
	-.1884615		-.2048718		-.2215385		-.2385641				12
	-.1852012		-.2007030		-.2162943		-.2320521				13
	.9827004		.9796517		.9763283		.9727034				14
	19.843		19.905		19.973		20.047			kip	15
	.0986		.0999		.1017		.1040			ft.	16
	100.8012		102.0558		103.9477		106.4886			ft.	17
	-18.6685		-20.4829		-22.4833		-24.7109		η_{10} = -1.6071	ft.	18
	99.0574		99.9791		101.4871		103.5818		ξ_{10} = 1.9131	ft.	19
	-364.036		-399.417		-438.424		-490.763		Σ = -40.238	ft. kip	20
									corr. = .0402		
DISPLACEMENTS											
	-3.6320		-3.9520		-4.2770		-4.6090		use corr. = .0430	kip	21
	-.1862564		-.2026666		-.2193333		-.2363590				22
	-.1831073		-.1986285		-.2142406		-.2300211				23
	.9830926		.9800751		.9767809		.9731852				24
	19.835		19.896		19.964		20.037			kip	25
	.0986		.0999		.1017		.1040			ft.	26
	100.8012		102.0558		103.9477		106.4886			ft.	27
	-18.4574		-20.2712		-22.2698		-24.4946		η_{10} = .4729	ft.	28
	99.0969		100.0223		101.5341		103.6331		ξ_{10} = 1.9009	ft.	29
	-359.920		-395.292		-434.261		-486.406		Σ = .456	ft. kip	30
DISPLACEMENTS											
	-3.6325		-3.9525		-4.2775		-4.6095		corr. = 0.0005	kip	31
	-.1862821		-.2026923		-.2193590		-.2363846				32
	-.1831317		-.1986526		-.2142645		-.2300448				33
	100.8012		102.0558		103.9477		106.4886			ft.	34
	-18.4599		-20.2737		-22.2723		-24.4971		η_{10} = .4486	ft.	35
	99.0965		100.0218		101.5336		103.6326		ξ_{10} = 1.9011	ft.	36
	-359.968		-395.336		-434.310		-486.458		Σ = -.017	ft. kip	37
									corr. = -.000017 neglect		

With a rough preliminary calculation as a guide, a value of 19.5 kips was assumed for H_p. The shears in the corresponding simply-supported beam, based on the original position of the load points, are shown on line 11. A better guess could have been made by anticipating the cable movements. Since the live load P is located eccentrically, the cable would be expected to shift toward the left, thus increasing the vertical reaction at C_0.

TABLE 5.—*CONTINUED*

WITH H_p = 20.195 KIPS

	C_6 – C_7	C_7 – C_8	C_8 – C_9	C_9 – C_{10}			
	-3.6453	-3.9653	-4.2903	-4.6223		kip	1
	-.1805051	-.1963506	-.2124437	-.2288834			2
	-.1776345	-.1926717	-.2078060	-.2231138			3
	.9840968	.9812636	.9781697	.9747924			4
	20.521	20.580	20.645	20.717		kip	5
	.1028	.1042	.1060	.1084		ft.	6
	100.8054	102.0601	103.9520	106.4930		ft.	7
	-17.9065	-19.6641	-21.6018	-23.7601	η_{10} = .0010	ft.	8
	99.2023	100.1479	101.6827	103.8086	ξ_{10} = -.0011	ft.	9
	-361.622	-397.116	-436.249	-479.829	Σ = .026	ft. kip	10
DISPLACEMENTS							
	-3.64533	-3.96533	-4.29033	-4.62233	corr. = -.00003	kip	11
	-.1805065	-.1963520	-.2124451	-.2288848			12
	-.1776358	-.1926730	-.2078073	-.2231151			13
approximation						ft.	14
	-17.9066	-19.6642	-21.6020	-23.7602	η_{10} = -.0004	ft.	15
	99.2022	100.1478	101.6827	103.8085	ξ_{10} = .0013	ft.	16
	-361.625	-397.119	-436.252	-479.843	Σ = -.014	ft. kip	17
					corr. = -.000014 neglect		

WITH H_p = 20.195 KIPS

C_6	C_7	C_8	C_9	C_{10}		
82.9326	65.0260	45.3618	23.7598	-.0004	ft.	18
-13.1174	-19.1490	-18.9382	-12.5402	-.0004	ft.	19
595.1575	694.3597	794.5075	896.1902	999.9987	ft.	20
4.8425	5.6403	5.4925	3.8098	.0013	ft.	21

The computations on lines 12 through 19 follow as before. The moment about C_{10} is 40.238 ft-kips, and the resulting shear correction is 0.0402 kip. However, a larger shear correction (0.0430 kip) was used, based on the experience of Example 1, with the intention of hastening the convergence. The new shears are listed on line 21.

At the close of the second cycle the moment about C_{10} is smaller but of opposite sign. This indicates that too large a positive shear correction was used on line 21.

In the third approximation to displacements the computation for Δl was omitted and the value of l_p from the second approximation was used. A glance at Example 1 will show that the change in Δl may normally be neglected for approximations beyond the second. The displacements of the cable end at the conclusion of the third cycle are shown on lines 35 and 36. The shear correction computed on line 37 negligible. The displacements at C_{10} and the shear

TABLE 5

	C_0	C_1	C_2	C_3	C_4	C_5
				FIRST APPROXIMATION TO DISPLACEMENTS		
1 V_p (from Fig. 6)	6.2817	5.9497	5.6247	5.3047	-3.0123	-3.3283
2 $\tan \psi$	.3110522	.2946125	.2785194	.2626739	-.1491607	-.1648081
3 $\sin \psi$	.2970153	.2826031	.2683070	.2540555	-.1475285	-.1626145
4 $\cos \psi$	.9548728	.9592366	.9633332	.9671897	.9890574	. .9866899
5 T_p	21.149	21.053	20.963	20.880	20.418	20.467
6 Δl	.1112	.1086	.1065	.1051	.1017	.1020
7 l_p	106.4958	103.9546	102.0624	100.8077	100.1797	100.1800
8 G_p	31.6309	29.3778	27.3841	25.6108	-14.7794	-16.2907
9 S_p	101.6899	99.7171	98.3201	97.5002	99.0835	98.8466
10 V_pS_p	638.785	593.287	553.021	517.209	-298.469	-328.991
					SECOND APPROXIMATION TO	
11 V_p	6.28167	5.94967	5.6247	5.30467	-3.01233	-3.32833
12 $\tan \psi$	.3110507	.2946110	.2785179	.2626724	-.1491621	-.1648096
13 $\sin \psi$	.2970140	.2826018	.2683056	.2540541	-.1475299	-.1626159
14 l_p				Assumed same as in first		
15 G_p	31.6307	29.3778	27.3839	25.6106	-14.7795	-16.2909
16 S_p	101.6900	99.7171	98.3201	97.5002	99.0835	98.8466
17 V_pS_p	638.783	593.284	553.018	517.206	-298.472	-328.994
					DISPLACEMENTS OF LOAD POINTS	
18 ΣG_p	0.0	31.6307	61.0085	88.3924	114.0030	99.2235
19 $\eta = \Sigma G_p - e$	0.0	-4.6693	-3.2915	4.2174	17.9530	-.7765
20 ΣS_p	0.0	101.6900	201.4071	299.7272	397.2274	496.3109
21 $\xi = \Sigma S_w - \Sigma S_p$	0.0	-1.6900	-1.4071	.2728	2.7726	3.6891

in segment C_0C_1 are plotted against the assumed value of H_p (19.50 kips) in Fig. 3.

The procedure was repeated for assumed values of H_p equal to 20.50 and 20.22 kips. Curves of η_{10} and ξ_{10} versus H_p intersect the zero displacement axis at H_p equal to 20.195 kips, for which the value of V_p for segment C_0C_1 is 6.2817 kips.

With H_p taken as 20.195 kips and V_p for segment C_0C_1 taken as 6.2817 kips, the procedure was repeated as shown in Table 5. At the conclusion of the second cycle the shear correction is negligible. The displacements of the cable

right end are shown on lines 15 and 16. These are sufficiently close to zero so that the problem may be considered solved. The displacements of the load points are listed on lines 19 and 21.

EFFECTS OF TEMPERATURE CHANGE

Displacements of the cable resulting from change in temperature can be determined in the same manner as displacements resulting from change in load.

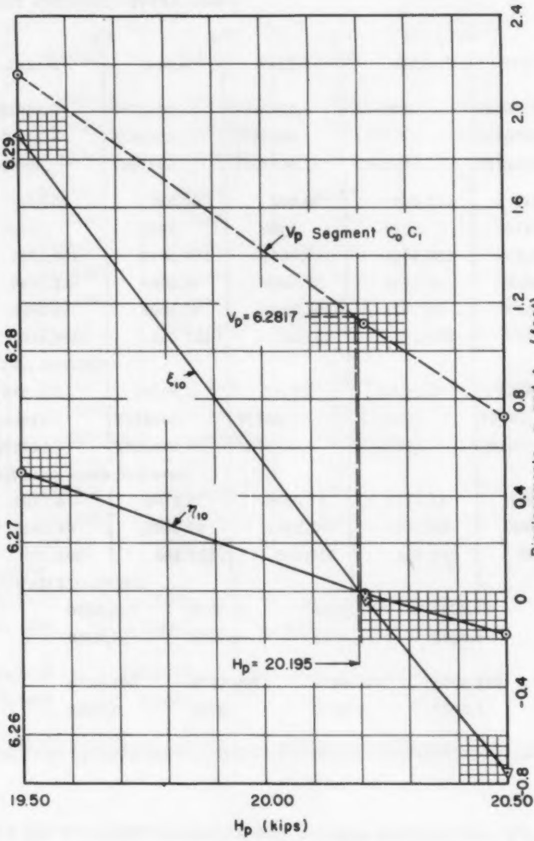


FIG. 3.—PLOT OF η_{10}, ξ_{10}, AND V_p VERSUS H_p FOR EXAMPLE 2

Given any position of a cable, the identical procedure is applied, but to the value of Δl in Eq. 5 must be added

$$\Delta l = \epsilon \, t \, l_w$$

in which t is the change in temperature in degrees Fahrenheit and ϵ is the coefficient of expansion.

APPENDIX.—NOTATION

The following notation (see also Fig. 1) is used:

A = cross-sectional area of cable.

C = load point designation.

e = vertical distance between the cable and the chord connecting the ends of the cable.

E = modulus of elasticity of the cable.

G_w = vertical projection of cable segment in the initial position. Positive when right end is below left end of segment.

G_p = vertical projection of cable segment in displaced position. Positive when right end is below left end of segment.

h = difference in elevation between cable ends.

H_w = horizontal component of cable tension in initial position.

H_p = horizontal component of cable tension in displaced position.

l_w = length of cable segment in initial position.

l_p = length of cable segment in displaced position.

L = span of cable.

M_s = simple-beam bending moment.

P = concentrated load causing cable movement.

S_w = horizontal projection of cable segment in initial position.

S_p = horizontal projection of cable segment in displaced position.

t = temperature change in degrees Fahrenheit

T_w = tension in cable segment in initial position.

T_p = tension in cable segment in displaced position.

u = vertical distance from the X-axis to the chord connecting the ends of the cable.

V_w = vertical component of cable -segment tension in initial position.

V_p = vertical component of cable -segment tension in displaced position.

w = weight of the cable per unit length.

W = concentrated load on cable in initial position.

ϵ = coefficient of thermal expansion.

η = vertical displacement of point on cable. Positive sign indicates movement in positive direction of y, downward.

θ = angle of cable segment with horizontal in initial position. Positive angle measured clockwise from the horizontal to cable segment.

ξ = horizontal displacement of point on cable. Positive sign indicates movement in negative direction of x, to the left.

ψ = angle of cable segment with horizontal in displaced position. Positive angle measured clockwise from horizontal to cable segment.

Journal of the

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

CONCEPTS OF STRUCTURAL SAFETY

By C. B. Brown[1]

SYNOPSIS

This paper presents methods of providing engineering safety and additional safety necessary for social purposes. The methods considered are the ratio, probability, and combined ratio and probability. Functional and collapse failure are discussed and also the concept of structural life in design.

INTRODUCTION

From the engineering viewpoint, if it were possible to build a structure exactly as designed, with all the materials of definite strength, only subjected to loads within those used in the design, and with the analysis and all assumptions of design correct, then it would not fail, and there would be no need to consider providing additional strength for the purpose of safety. In fact, all these factors are variables and few can be accurately predicted. To provide for this absence of certainty the strength of the structure is increased. This increased strength provides a certain level of safety against extreme conditions of the variables.

The final level of safety is composed of that provided for (a) engineering purposes and (b) social consequences of failure. The additional strength required for (a) is for the purposes of the previous paragraph; that for (b) is determined by taking into account the financial cost, inconvenience and risk of death and injury associated with failure.

Note.—Discussion open until May 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 12, December, 1960.

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Structures of different levels of safety can be built. The level provided depends largely on the amount of money available. As the safety increases, the cost of the structure also goes up, because, to ensure additional strength, either larger sections are required with magnification of the foundation, or stronger and more expensive materials are required. A structure in which failure is nearly impossible can be constructed, but the cost would prove prohibitive. In practice, a lower standard of safety is usual, and is specified by a committee of experts, or the designer, or the owner. The methods of specifying safety must provide a solution to the engineering and social problem.

Engineering structures fail when they collapse or show functional deficiency. Collapse occurs when the applied loads exceed the capacity of the structure to resist them at the critical time. This situation results from the inability to determine the applied loads, the strength capacity of the structure, or both in the design, or some error in construction. Functional deficiency occurs when conditions limit the use of the structure under design loads; it includes excessive resonance, deflection, and deformation. The solution to the engineering problem requires reasonable methods of specification which limit the chance of structural failure.

Additional to the specification of engineering safety, is the social problem. The solution of this requires the provision of an additional level of safety, above that required by engineering considerations. This additional level protects the owner, the user, and the general public from the consequences of failure. The owner has a financial, if no other, interest in the structure; the user has a risk of injury or death associated with collapse, and the public may lose confidence in a particular type of structure.

STRUCTURES AND FAILURES

A structure may be defined as a member or members arranged and proportioned to resist the applied loads, and to behave functionally. Generally a structure is made up of a group of members, which, by their individual action affect the strength of the group. Thus, members without the structure can affect the load on the structure, but not the strength.

Although all members and parts of a structure must affect the ultimate, and functional strength, the tendency in design has been to ignore the action of certain parts whose effect was difficult or impossible to evaluate. This has led to confidence in design methods because of the success of the structure. This success was often due to the help of the parts of the structure not considered in design, and not necessarily to the veracity of the design methods. R. H. Wood[2] indicates that the safety level of city buildings against indefinitely large elastic displacements must have been very low, but the apparent success of the structures must have been due to the action of the brick and concrete cladding, not considered in the original design. Similarly, the use of part of the structure for design purposes has led to disagreement between calculated and actual performance. L. T. Oehler[3] found that the actual deflections of bridges tested was only a small part of that computed. This, in floor systems designed for noncomposite action, possibly reflects the partial composite action between the

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2 "The Stability of Tall Buildings," by R. H. Wood, Proceedings, Institute of Civil Engineering, Vol. 11, Sept. 1958.

3 "Vibration Susceptibilities of Various Highway Bridge Types," by L. T. Oehler, Proceedings, ASCE, Vol. 83, No. ST 4, July, 1957.

steel stringers and concrete deck providing stiffer longitudinal beams, and lateral interaction of the whole deck system. In Europe, bridge designers have endeavoured to make use of the whole floor system in resisting loads.[4,5] This has led to the complication of the design computations, together with reduction in structural weight and cost, but has produced results in which the observed and theoretical deflections are in close agreement.[6] Until recently, safety was therefore considered by two standards; in design, the safety of the member or part was paramount, whereas, in use, the safety of the whole structure was the basic concern. Today, efforts to consider the whole structure from the safety viewpoint at all stages has received attention and this attitude is supported by authoritive opinions in America and Great Britian.[7,8] In this paper ultimate failure of the whole structure is considered, and member failure in indeterminate structures, which does not lead to collapse is considered as functional failure.

DESIGN AND CONSTRUCTION

The work of design involves the provision of a structure with a decided level of safety at a minimum cost. The steps are as follows:

(1) Determination of design loads.
(2) Determination of the strength of the structure.
(3) Determination of a quantitative value for safety.

Step (3) provides for the inadequacy of knowledge in steps (1) and (2) and any social safety level required. Step (2) includes

(a) Consideration of material strength.
(b) Selection of sections and structural arrangement.
(c) Analysis of resistance of the structure in the light of (a) and (b).

The structure when finished should be the same as designed. Any variation affects steps (2) and (3).

Generally, step (1) has received less attention than (2) and the design loading is often the weakest part of the design procedure. Efforts have been made to determine the load intensities, patterns, frequencies of occurrence, and durations.[9,10,11,12,13] These investigations require extensive data in order to deal

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4 "New Ways to Cut Bridge Weight Load for Record Spans," Engineering News Record, November, 1957.

5 "Orthotropic Plate Design for Steel Bridges," by R. Wokhuk, Civil Engineering, Vol. 29, No. 2, February, 1959.

6 "Discussion on Live Loading for Long-Span Highway Bridges," by L. Balog, Transactions ASCE, Vol. 119, 1954.

7 Synopsis of First Progress Report of the Committee on Factors of Safety, by O. G. Julian, Proceedings, ASCE, Vol. 83, No. ST 4, July, 1957.

8 Report on Structural Safety, Journal of the Institute of Structural Engineers, Vol. 24, No. 5, May, 1955.

9 "Live Loading for Long-Span Highway Bridges," by R. J. Ivy, T. Y. Lin, S. Mitchell, N. C. Raab, V. J. Richey, and C. F. Scheffey, Transactions, ASCE, Vol. 119, 1954.

10 "Highway Bridge Live Loads Based on Laws of Chance," by H. K. Stephenson, Proceedings, ASCE, Vol. 83, No. ST 4, July, 1957.

11 "Survey of Live Loads in Offices," by C. M. White, Steel Structures Research Committee, First Report, 1931.

12 "Design Live Loads in Buildings," by J. W. Dunham, Transactions, ASCE, Vol. 112, 1947.

13 "Wind Loads on Structures," by M. R. Horne, Journal of the Institute of Civil Engineering, No. 3, 1949-50, January, 1950.

with the problem on a statistical basis, and these are difficult and expensive to obtain. The problem here dealt with is the methods of specifying structural safety, that is the basis of step (3).

ENGINEERING METHODS

A method of specifying safety from the engineering aspect must attempt to avoid the excess of the applied loads over the resistance of the structure. Curve L on Fig. 1 shows the different magnitude of loads that an idealized structure may be subjected to, plotted against the relative frequency of occurrence. Curve S indicates the variability of strength magnitude of a large number of structures designed and constructed to the same requirements. Failure occurs where the two lines intersect, and where any value of L is in excess of S for a particular structure. Under present conditions there is always a range of loads and strengths, and whatever arrangements of the S and L curves exist the tails will intersect, implying a possibility of failure. The curves can be moved apart thus reducing the area of interference but intersection will occur. Only when the high side of the L curve and the low side of the S curve are vertical lines can safety be ensured. With present day knowledge this does not occur. Under these circumstances no structure can be completely safe, and there is always a chance of failure.

Attempts at providing safety for these typical curves can be made by

(1) Specifying as a minimum a ratio $\frac{S_1}{L_1}$, known as the factor of safety. The term L_1 is the specified maximum design load anticipated under working conditions, and S_1 is the resistance of the structure arrived at by specifying the variables in design.

(2) Specifying as a maximum the probability of failure.

(3) Specifying as a minimum a factor of safety coupled with a maximum probability of failure.

Specification of a factor of safety alone does not define the probability of failure. This is indicated in Fig. 2. In Fig. 2 (a) the loads and strength variables are closely predictable, and the probability of failure is small. In Fig. 2 (b) the reverse is the case. With the same factor of safety, the condition in Fig. 2 (a) has a smaller probability of failure than Fig. 2 (b).

The values specified for functional failure and collapse vary. In general a greater risk of functional failure than collapse is acceptable.

SOCIAL CONSEQUENCES OF FAILURE

The application of additional safety, above that required for engineering purposes, to provide for social consequences of functional or collapse failure is by the varying of values specified (ratio, probability, and ratio and probability).

The social consequences include the financial loss to the owner, loss of life, injury, damage to human society, interference with use, repairs and reconstruction costs resulting from failure. S. O. Asplund[14] suggests that these factors should be included as a capitalized cost, together with the initial cost, and the costs of maintenance and repair, in the determination of the actual cost

14 "The Risk of Failure," by S. O. Asplund, Structural Engineer, Vol. 36, No. 8, August, 1958.

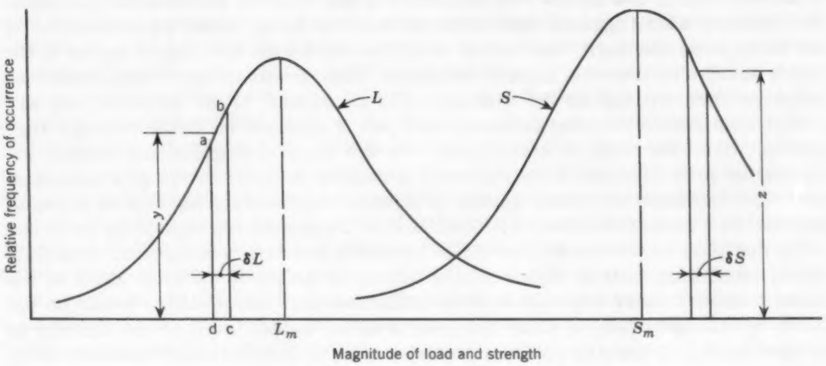

FIG. 1

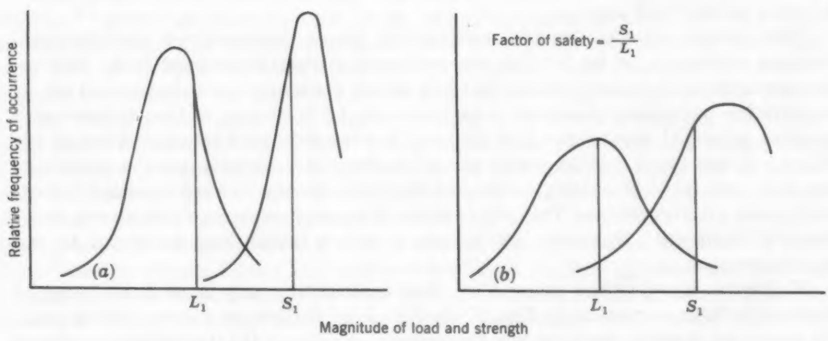

FIG. 2

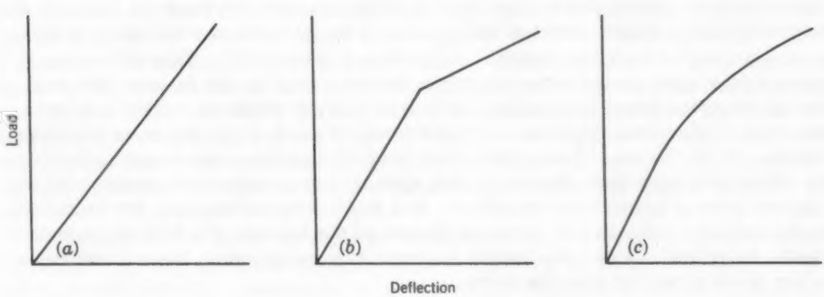

FIG. 3

of the structure, and in the determination of the level of safety to be provided. Mr. Asplund also suggests that in the decision of safety level, consideration of the structures not built, but which could be built with the money saved if the safety level was lowered, should be made. This involves the cost in lives and inconvenience caused by the absence of structures. These implications are social, administrative, and political, and yet, if considered in the manner suggested, affect the cost of structures. As the work of engineering design involves the provision of a structure with a decided level of safety at a minimum cost, the implications would appear to have an engineering as well as a social aspect. The consideration of all these social consequences cannot be achieved in the decision on the safety level to be provided in a single structure. Possibly the national committees which decide levels of safety in various parts of the world consider these aspects in their deliberations. An instance would be the safety levels for bridges. Here the effects of the safety level on the number of bridges built in a country could be considered. The level of safety decided being that which provides the maximum economic gain to the community. For instance, the value of 1,000 bridges built to a level of safety contrasted with 1,100 cheaper bridges built to a lower level may have to be decided, taking into account the cost of the inconvenience, death, and injury associated with the 100 unbuilt bridges in the first case.

The method of providing additional safety, above that required from the engineering viewpoint, is by varying the ratio and probabilities specified. The increase in safety level may be applied to a whole structure, or to individual parts, or only for particular methods of failure. A. L. L. Baker, in the discussion of another paper,[21] states that it is illogical to have different factors of safety for failure of the steel and the concrete in reinforced concrete; also, a structure has only one level of safety in use, and that this should be kept constant for all parts, and all structures. This attitude would appear reasonable from the engineering viewpoint. However, variations in safety levels may be specified for social reasons.

K. Hajnal-Konyi[15] has pointed out that a structure may have three types of load-deflection curves as in Fig. 3. In Fig. 3 (a) failure is sudden with a brittle material obeying Hooke's law throughout. In Fig. 3 (b) the ductile material shows sudden permanent deformation with little increase in load beyond the yield point. In Fig. 3 (c) the tough material with limited linear load: deflection characteristic, shows increasing rate of deflection with increase of load. It is suggested that a higher level of safety should be introduced when there is little or no warning of incipient failure, as in Fig. 3 (a) and (b). This difference in safety could apply to the same member, as instanced by the failure of a reinforced-concrete beam in bending. A tensile failure would be proceeded by indications of distress, whereas a compression failure of the concrete would be sudden. F. G. Thomas[16] suggests, that certain members are more critical to the whole structure than others. In this case, the more critical members merit a higher level of safety than the others. In a multi-floored building, for instance, the failure of a beam would not be as tragic as the failure of a column; it would appear reasonable from the social viewpoint to ensure that the column has a higher level of safety than the beam.

In use, varying levels of safety are specified, presumably after consideration of the social consequences of failure. However, in the discussion by S. C. C.

15 "Reinforced Steel in Concrete and the Concept of Safety," by K. Hajnal-Konyi, Proceedings, Am. C. I., Vol. 48, 1952.

16 "Load Factor Methods of Designing Reinforced Concrete," by F. G. Thomas, Reinforced Concrete Review, Vol. 3, No. 8, 1955.

Bate,[17] it is suggested that the factor of 2 when primary failure is in tension, and 2.5 when in compression, in beams, as often specified for prestressed concrete work, was because there is a greater variance in concrete than in steel strength. This depends upon the method of selecting material strength, and as the same discusser suggests, the different variances should not be reflected in the safety levels specified, but in the decision of the material strength to be used in design. A consistent attitude would appear the most reasonable, and the design strengths used in a structure should be such that the percentage of test results below the selected design value are the same for all materials.

In the decision on the social level of safety to be provided, use of judgment appears essential as it is impossible to determine the numerical effect of the social consequences of failure on the probability or risk of failure. Much of the social consequences of failure can be expressed as a cost. If some of this amount is introduced into the cost of the structure, the decision on the safety level becomes an engineering consideration.

RATIO METHODS

This is the traditional method of applying safety. A. G. Pugsley[18] and D. T. Wright,[19] give a historic account of the use of this method. Until the 19th century safety consisted in the success, or otherwise, of the structure in use. The only idea of the level of safety in a type of structure was possible when other similar structures with different sections were built, and the greatest sections to just fail were determined. Only then could it be decided that the size first used was a definite amount larger than just necessary. At the same time dead loads were dominant in permanent structures of masonry, and if the collapse did not occur at the striking of the centering, it was unlikely to later, except when due to differential settlements. With cast iron, live loads became more important and factors of safety were provided, and defined as the ratio of collapse load to design load. The collapse load was decided by physical tests on columns and beams, the design load was considered as the greatest load the structure would be called on to carry in use. Both these values were variable, the collapse load because of non-uniformity of the strength of castings, and the design load because of the dynamic affects of movement of the loads. The variabilities were accounted for in the factor of safety. This method was continued with the introduction of mild steel and wrought iron. Model tests to determine the various levels of safety of different types of structure were popular. The load factor method, that is, the ratio of collapse to design load, was used in these tests by Stephenson on the Brittannia Bridge, and Hope's proposed crossing of the Hudson. With mild steel the excessive permanent deformations without collapse were dealt with by the use of proof loads; the load factor became the ratio of the proof load to design load. Proof load was either the load at which permanent deformations commenced or the load the structure was tested to, prior to use.

The beam theory became popular when steel was in general use, and the stress factor, defined as the ratio of ultimate stress of the material: the greatest

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17 Discussion on "Prestressed Units for Short-Span Highway Bridges," by S. C. C. Bate, Proceedings, Institute of Civil Engineering, Part 2, June, 1955.

18 "Concepts of Safety in Structural Engineering," by A. G. Pugsley, Journal of the Institute of Civil Engineering, Vol. 36, No. 5, March, 1951.

19 "The Evaluation of Highway Bridges," by D. T. Wright, Ontario Joint Highway Research Programme, Dept. of Civil Engr., Queen's College, Kingston, Report Q6-1.

stress induced by design loads into the material (design stress) was used. The ultimate stress was the yield stress and deformations were recoverable. Because of the slenderer sections possible with the greater material strength of mild steel, collapse by instability within the elastic range was possible. With this problem, load factor methods would have been advisable, but the stress factor was so popular that it was used irrespective of its merits.

With reinforced concrete, many structures were built under the impression that concrete tensile properties were improved when the concrete was reinforced.[20] By the time this idea of Considere was refuted, reinforced concrete was a proven structural form, and it was natural that the popular stress factor should be instated as the safety measure in this field. Most of the concern of the time was about the allowance of cracks in the concrete cover of the tension steel. Prohibitive specifications of a factor of 1.5 to 2.5 against cracking caused the early experiments in prestressing concrete to be carried out. With the use of prestressed concrete the stress factor was installed as the basis of safety. However, in recent years the load factor has been specified in codes of design practice for steel, reinforced concrete and prestressed concrete structures. Functional failure is dealt with either by the use of stress factors for design loading or the use of proof loads with load factors. After cracking, prestressed concrete ceases to act elastically. The resultant non-linear stress: load curve allows more reasonable levels of safety with the use of load factors. In reinforced concrete the stress: load curve is not linear, and the structures are usually monolithic, and thus highly redundant. In these circumstances major stress redistributions take place. This extra resistance and the non-linearity of the stress: load curve make the load factor approach realistic. The present use of the load factor in steel allows the use of plastic and elastic plastic analysis methods, which take account of the redistribution of moments in indeterminate structures.

The ratio known as the factor of safety, v, is defined as the ratio of the resistance of the structure, S_1: the design load effect, L_1. In the stress factor the value S_1 is the maximum stress in the critical section at failure, in the load factor the value S_1 is the load applied just to cause failure. In the stress factor L_1 is the maximum stress produced at the critical section by the design load, in the load factor L_1, is the design load. The design load is generally the greatest reasonably anticipated load in use. However, the value L_1 may be selected in any position on the L curve in Fig. 1. The resistance, S_1, is generally the lowest reasonably anticipated strength of the structure, but may be selected in any position on the S curve in Fig. 1. The selection of lower values of L_1 and higher values of S_1 give apparently higher values of v. In fact, the risk of failure with the unchanged L and S curves is the same, and the apparently high value of v is not the intention of the ratio method as interpreted by the writer. The intention is considered to be to give a ratio of the lowest reasonable anticipated resistance of the structure: the highest reasonably anticipated loading. This is the view taken by A. Baker.[21]

From the preceding

$$v = \frac{S_1}{L_1} > 1 \text{ for safety} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

20 "Prestressed Concrete Design and Construction," by F. Walley, Her Majesty's Stationary Office Publication.

21 "The Work of the European Committee on Concrete," by A. L. L. Baker, Structural Engineering, Vol. 36, No. 1, January, 1958.

The values of S_1 and L_1, obtained by decision of the design variables may not be complete. If L_1 is multiplied by a factor, i, which varies according to the certainty and adequacy of the loading data and S_1 is divided by a factor, j, which varies according to the certainty and adequacy of the strength data of the structure, then S_1/j is the strength necessary to balance i L_1. Then

$$i\ L_1 = \frac{S_1}{j} \quad \text{.......................} \quad (2a)$$

$$\frac{S_1}{L_1} = i\ j \quad \text{.......................} \quad (2b)$$

and

$$v_1 = i\ j \quad \text{.......................} \quad (2c)$$

in which v_1 is the factor of safety from the engineering viewpoint.

The social factor of safety is satisfied by increasing the provided strength, S_1, by a factor so that $S_1/(j\ k)$ is the strength necessary to balance i L_1. Then

$$i\ L_1 = \frac{S_1}{j\ k} \quad \text{.......................} \quad (3a)$$

$$\frac{S_1}{L_1} = i\ j\ k \quad \text{.......................} \quad (3b)$$

and

$$v_2 = i\ j\ k \quad \text{.......................} \quad (3c)$$

in which v_2 is the final factor of safety. The values of i, j and $k > 1$ for normal selections of reasonable loads and strengths.

REDUCTION OF v, FACTOR OF SAFETY, WITH DIFFERENT LOAD ORIGINS

In Eq. 1 the values of S_1 and L_1 may vary. The strength of a structure is time dependent. During its life fatigue, corrosion, creep, wear and the reduction of load capacity with time of loading will reduce the strength of the structure, and the increase in material strength with time may increase the strength of the structure. This means that different values of S_1 may be selected for different episodes in the life of a structure. In fact, this is seldom done and S1 is taken at a pessimistic level. Exceptions occur, such as in pretensioned prestressed concrete where transfer of the prestress occurs soon after casting, and the value S_1 is lower than that used when the concrete in the member is fully matured, and in use.

In the case of L_1, the loads are from different sources. The only load of profit to a structure is the live load, but at all times the dead load also must be carried. The two together are termed the working load. Additional loads of no profit to the structure have to be resisted. These loads are of natural

origin, such as wind, earth quakes, snow and ground deformation, or of human origin, such as, artificial flooding and bomb blast.

Only the dead load is certain of occurring; if the structure is put into use as intended the live load will operate, possibly intermittently. The live load and all non-working loads will combine with the dead load when they occur. These non-working loads have not a great fatigue effect, and the permanent deformation to the structure can be limited if the number and intensity of occurrence in the structures life-time can be foretold. Under these loadings, combined with the dead load, a lower value of v than that used with working loads is possible.

The high intensity non-working loads are found to have a smaller chance of occurring than the working loads. The probability of any load occurring has a value which may possibly be determined. The probability of it occurring with the dead load is the same value. The probability of it occurring with the dead load and any other load is smaller, and the more loads considered the smaller the probability of occurrence together. It would appear reasonable to reduce the value of v as more loads are considered. If P_1, P_2, P_3 . . . . . P_n are the separate probabilities of each loading occurring then the probability of them all occurring together, P

$$P = P_1 \cdot P_2 \cdot P_3 \cdots\cdots P_n \qquad \text{(4)}$$

Therefore, the probability of the occurrence of any number of loads together is the product of their individual probabilities of occurring. Some qualitative help in deciding the reduction in v for different load combinations can be obtained from this simple statistical analysis if enough data to estimate the probabilities are available.

Two reasons for reducing the safety factor under certain conditions are shown. The first is due to different chance of damage from various load sources, the second is due to the different probability of individual loads occurring, and combinations of loads occurring. The strength value in Eq. 1 usually remains constant except when some particular change in strength with time is being allowed for. The result of the different damage effects of loads can be provided for by lowering the value of v with the less damaging types.

$$v_a \left(L_a + L_b\right) = S_1 \qquad \text{(5a)}$$

and

$$v_b \left(L_a + L_c\right) = S_1 \qquad \text{(5b)}$$

in which $v_a > v_b$ and L_b is more damaging than L_c, and L_a is the dead load. Similar adjustments for v can be made to allow for less probable load combinations. The Eq. 5 may be applied to this, where $v_a > v_b$ and L_b is the more probable loading than L_c, and L_a has a probability of occurrence of unity. With combinations of more load origins

$$v_c \left(L_a + L_b + L_c + L_d\right) = S_1 \qquad \text{(6a)}$$

and

$$v_d \left(L_a + L_c + L_e + L_f\right) = S_1 \qquad \text{(6b)}$$

in which $v_c > v_d$ and the v_c combination of loads is more probable together than the v_d. In this case the chance of the loads occurring together is not dependent upon the number of load origins involved, but on the combined probability of occurrence. Therefore, the v_c combination may have more or less load origins than the v_d, but the v_c combination will be more probable.

An additional cause of varying v is the value of i selected in Eqs. 2 and 3. Fig. 4 shows two curves of load magnitude: relative frequency of occurrence. In the curve in Fig. 4 (a) the load falls within narrow limits and would be accurately forecast. This could apply to dead load or static water pressure and the value of i would approach unity. In the curve in Fig. 4 (b) the load varies over a wide range and the forecasting at high magnitudes is difficult. The value of i depends upon the certainty of the selection of the design load. If a known maximum type load L" with only a very small number of occurrences of greater intensity exists, or if the load can be physically limited, then the value i will be small. If the load selection is less certain, in the region of L' as opposed to L", the value of i will be larger. The value of i affects v in the same manner. Then

$$v_e L_a + v_f L_b = S_1 \quad \text{(7a)}$$

$$v_e L_a + v_f L_b + v_g L_c = S_1 \quad \text{(7b)}$$

Here v_e, v_f, and v_g need not be the same, but depend on the value of i for each loading.

Rüsch[22] is reported as favouring the selection of highway loadings as the maximum anticipated, thus ensuring that the value of v is the same as for dead load. This method has been popular in the highway regulations from all parts of the world in the past. Mr. Wright[19] has indicated the increasing actual factor of safety as the dead load to total load ratio increases, if the load value for v approaches unity. He suggests the use of different values of v as suggested previously. This has proved to be acceptable and many specifications allow this type of provision. The combination of loads as curve (a) with curve (b) in Fig. 4, will produce a sum of loading: relative frequency curve as curve (c), with smaller load variation than (b). In this case a value of (v) to apply to the two loadings would be more than that for (a) but less than for (b). However, this value of v would vary for all ratios of Load curve (a) to Load curve (b). It would appear easier to select v on the individual loadings as in Eq. 7. If the selection of L_b is as suggested by Rüsch then $v_e = v_f$.

The combination of these variables, that is, risk of certain damage forms with different loads, different chances of loads and load combinations occurring and the varying values of all loads, can be expressed by varying the values of v for all loads, and for all load combinations. Thus numerically we may have

$$1.1\ L_a + 2.0\ L_b = S_1 \quad \text{(8a)}$$

$$1.1\ L_a + 1.0\ L_b + 1.5\ L_c = S_1 \quad \text{(8b)}$$

22 "Fatigue Resistance of Prestressed Beams in Bending," by C. E. Ekberg Jr., R. E. Walther, and R. G. Slatter, Proceedings, ASCE, Vol. 83, No. ST 4, July, 1957.

and

$$1.1\ L_a + 1.0\ L_c + 1.0\ L_d = S_1 \qquad (8c)$$

The values of v being selected on the basis of these three variables.

STRESS AND LOAD FACTORS

This subject has been considered in part previously. For structures with critical sections with linear stress to load relationship the result is the same. With a non-linear relationship at critical sections the values of the load factor, v_L, and the stress factor, v_s, vary. A. Baker[23] has classified structures as appreciating and depreciating. Appreciating structures are those in which the stress at critical sections increases less rapidly than the load, and depreciating structures are those in which the stress increases more rapidly than the load. These are illustrated in Fig. 5.

$$v_s = \frac{S_s}{L_s} \qquad (9a)$$

and

$$v_L = \frac{S_L}{L_L} \qquad (9b)$$

In the appreciating structure $v_L > v_s$ and in the depreciating type $v_L < v_s$. In the linear relationship $v_L = v_s$.

In the appreciating type of a structure the load factor gives the overload capacity but the low value of v_s with the possible high stresses at design load may lead to excessive deformations and functional failure. In the depreciating case the load factor gives the overload capacity but the stress factor gives an optimistic view of the additional structural strength above design conditions.

With the use of a stress factor, and defining collapse as when the yield stress or the modulus of rupture is attained at a critical section, indeterminate and simple structures fail at the same condition. It has been shown[24,25] that indeterminate structures redistribute moments from sections where the limiting stress condition has been reached to less highly stressed sections, failure ultimately occurring at a higher load than that which caused the initial limiting stress condition. To provide for this additional load carrying capacity in bending, load factors are used. Generally, in all structures the load factor gives a better picture of overload capacity than the stress factor.

THE PROBABILITY METHOD

This method has received considerable use within the aircraft industry by the employment of statistical means to analyze loadings and strengths. This is

23 "Reinforced Concrete Structures," by A. L. L. Baker, Structural Engineering, July, 1958.

24 "The Steel Skeleton," by J. F. Baker, M. R. Horne, and J. Heyman, Vol. 2, Cambridge Univ. Press, New York, N. Y.

25 "Redistribution of Design Bending Moments in Reinforced Concrete Continuous Beams," by A. H. Mattock, Proceedings, Institute of Civil Engineering, Vol. 13, May, 1959.

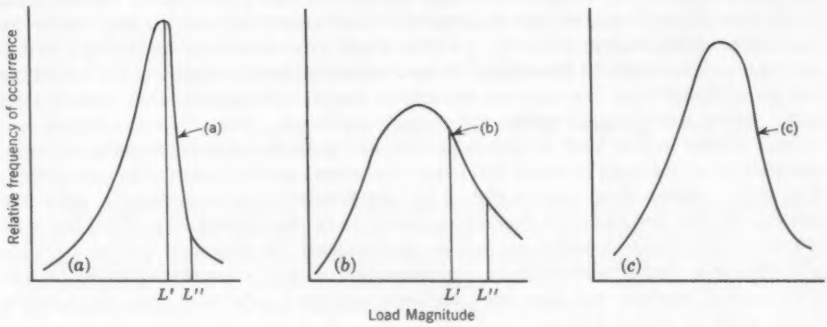

FIG. 4

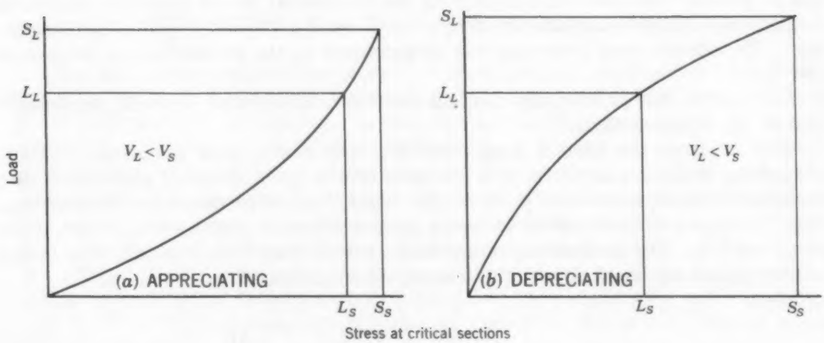

FIG. 5

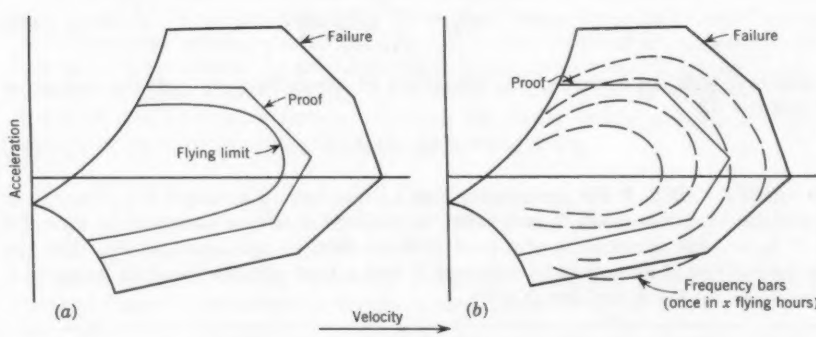

FIG. 6

indicated elsewhere.[18] Aircraft wings resist bending and torsion. The bending loads are proportional to acceleration normal to the wing and torsion loads are largely proportional to velocity. Test aircraft provided records of velocity and normal acceleration to the wing. From tests and design analysis the breaking and proof conditions for various velocities and accelerations were determined and a flying limit placed within the proof conditions. The proof conditions occurred either at the load at which permanent unacceptable deformations commenced, or at the load to which full scale tests had been satisfactorily completed. Fig. 6 (a), shows this, and in Fig. 6 (b) the frequency of occurrence bars are shown. By the extension of these frequency bars the probability of occurrence of proof or collapse conditions can be determined. In this way the probability of collapse or unserviceability is expressed as once in a number of flying hours. This type of method has also been applied to large production civil engineering units, such as railway ties.

In large and massive structures with long intended life and where the structure is unique, that is, no other one exactly the same will be envisaged, the use of probability methods is not easily possible. However, in the N. C. I. T. building in Paris[26] the steel ties accepting the horizontal thrust from the arches at each corner of the triangular building in plan, were designed by statistical analysis. The stress used in design was determined on the probability of failure of 10^{-5}.

Generally, the probability method has been illustrated by S. O. Asplund[14] and A. M. Freaudenthal.[27]

Fig. 1 shows the plot of magnitude of loads acting on a structure against frequency and the magnitude of a strength of a large number of structures designed and constructed alike to resist the loads, against frequency of occurrence. The frequency of occurrence of loads lying between L and $L + \delta L$ is the area abcd = $y\,\delta L$. The probability that a load greater than L will occur, P_L, is the area bounded by the L curve, the loading of magnitude L and infinity.

$$P_L = \sum_{L}^{\infty} y\,\delta L = \int_{L}^{\infty} y\,dL \quad \ldots\ldots\ldots \quad (10a)$$

as $y = f(L)$ then

$$P_L = \int_{L}^{\infty} f(L)\,dL \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10b)$$

similarly with the S curve, the frequency of strength lying between any value S and $S + \delta S$

$$S + \delta S = z\,\delta S = p \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

in which $z = f(S)$. If the probability that a structure of strength S will occur is p and the load necessary to just cause the failure of such a structure of strength S is L, and the probability of a load greater than or equal to L is P_L, then the probability of a structure of strength S and a load greater than or equal to L occurring on the structure is $p\,P_L$.

26 "The Design and Construction of the Shell Roof of the Exhibition Palace of the National Centre of Industries and Technology," by N. Esquillan, Paris, France, Cement and Concrete Assoc., London, England, December, 1958.

27 "Safety and the Probability of Structural Failure," by A. M. Freudenthal, Transactions, ASCE, Vol. 121, 1956.

The probability of all strengths of structures having loads on them greater than or equal to the failing load is the sum of all values of p P_L within all limits. This is the probability of failure, P_F. As P_L is defined in Eq. 10b and p = z δS = f(S) δS, then for all positive values of strength and loading

$$P_F = \sum_0^{\infty} P_L \, f(S) \, \delta S = \int_0^{\infty} P_L \, f(S) \, dS \quad \text{.........} \quad (12)$$

The variations in the safety level mentioned previously, can be handled in a similar manner in the probability method, as in the ratio method. With the working loads a probability of failure may be specified and for additional loading the specification of a more remote acceptable probability may be used depending upon the chances of the loading occurring together and the effects of the loadings.

The probability of failure may be dealt with by using the loads or the stresses caused by the loads as a basis. Generally, however, the method has been associated with loads.

COMBINATION OF RATIO AND PROBABILITY METHODS

The use of the combination of ratio and probability methods has been advocated by Asplund[14] and O. G. Julian.[7] In Fig. 1 the arithmetic means of L and S are indicated as L_m and S_m. In this method the minimum factor of safety to ensure that a given probability of failure P_F, of the structure is not exceeded is specified. The factor of safety can be defined as the ratio[7, 14] of S_m: L_m. Any values of S and L could be used, but these values would have to be defined. The value of the factor of safety can be adjusted by the variation of the strength. The probability of failure is defined in the previous article. This method avoids the pit fall of the ratio method, where small variations in S and L produce a small probability of failure and, with the same factor of safety, wide variations in S and L a high probability failure. In Fig. 2 (b) the probability of failure would control the safety level whereas in Fig. 2 (a) the ratio would be dominant. Between these cases balanced distribution will occur where the specified ratio and probability are reached together.

A method of associating load factors and probability of failure has been shown elsewhere.[24] The design load = L_1 and the load factor = v_L, therefore the structures are designed to resist a load v_L L_1. In fact, because of imperfections in use in each structure, the actual strength of any structure = R v_L L_1, where R varies. A curve similar to Fig. 1 for S can be drawn for R, where R v_L L_1 = S and v_L and L_1 are constants. By using Eq. 10 (b) Baker arrives at a probability of the load exceeding R v_L L_1 occurring of P_R

$$P_R = v_L \, L_1 \int_R^{\infty} F\left(M \, v_L \, L_1\right) dM \quad \text{.............} \quad (13)$$

in which M is the dummy variable. Similarly, using Eq. 12 the probability of failure when the structure is designed for a factor of v_L is defined by

$$P_F = \int_0^{\infty} P_R \, f(R) \, dR \quad \text{.............} \quad (14)$$

where the probability of strengths lying between $R\ v_L\ L_1$ and $\delta\ (R\ v_L\ L_1)$ is p = f(R) dR, and P_R was given previously. Practical examples of the application of ratio and probability methods in design are given by Hsuan - Loh Su.[28]

Eq. 14 gives a relationship in which a minimum value of v_L and maximum value of P_F may be specified. Also Baker points out that it may be the basis for a rational decision on values for load factors.

The discussions on the previous two methods apply to this method and the level of safety may be varied according to the anticipated structural damage of a loading and the chance of the loadings occurring singly or in combination. In this, and the probability method the variations in the values of L which cause the value of (i) to be varied in the ratio method are accounted for in the value of P_F employed.

VALUES OF SAFETY LEVEL

It is not the intention of this paper to deal with numerical values for the methods considered. All that will be mentioned here is the method of obtaining values.

In the ratio method statistical means are necessary to decide on reasonable values of the ratio for various loading and items as mentioned previously. In the decision on a general safety level, the safety factor may be associated with probability of failure as in Eq. 14. This would appear a reasonable way of obtaining a value with adequate data available. Another method is by testing of existing structures which appear satisfactory in use and determining the actual safety factors with respect to a definite material condition (such as a outer fiber stress). In this manner values for ultimate and functional safety factors could be correlated and the lower figures obtained could be suggestive of acceptable safety levels in similar structures for design purposes. The value of the ratio in use in design must be defined in terms of S and L. In this, the reasonable manner is to use a value of S associated with a definite probability of a lower strength occurring. Thus

$$S_1 = S_M - \alpha\,\delta_S \quad \dots\dots\dots\dots\dots (15)$$

in which S_1 is the strength value in Eq. 1; S_M is the mean strength value; δ_S is the standard deviation; and α is a value depending upon the probability of strengths lower than S_1 occurring. In the case of loads,

$$L_1 = L_M + \mu\,\delta_L \quad \dots\dots\dots\dots\dots (16)$$

in which L_1 is the load value in Eq. 1; L_M is the mean strength value; δ_L is the standard deviation; and μ is a value depending upon the probability of loads greater than L_1 occurring.

In the use of the probability method of specifying safety the probability may be decided by reference to existing acceptable values. The probability of death in a train or road accident to arrive at a value which apparently does not cause public concern has been used.[23,29] A. M. Freudenthal[27] uses an economic ap-

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28 "Statistical Approach to Structural Design," by Hsuan-Loh Su, Proceedings of the Institute of Civ. Engineering, Vol. 13, July, 1959.

29 "Theorie Probabiliste de la Securite," by M. Prot, Revue Generale de Chemins de Fer, June, 1951.

proach and arrives at probability of failure by ensuring that the sum of the initial cost of the structure and the cost of failure is a minimum. Asplund[4] suggests the same type of approach with the cost of death involved evaluated financially and included in the sum.

For any rational approach to safety it is essential to have adequate data of loading and structural strength, and to treat the data statistically in order that the information may be most usefully employed. The extent of the success of any method depends upon the information available. The information should be balanced so that any decision on loads is supported by an equivalent amount of evidence as that on which a decision on strength is made.

FUNCTIONAL FAILURE

The methods of providing functional safety are two fold. First, by altering the conditions of the three methods specified, and second, by specifying limits which must not be exceeded with design loads. In many cases, such as cracking and leaking in a water tower, functional failure may be of greater importance than collapse, whereas, similar cracking in a grain silo would not be as critical.

Alterations of the factor of safety in the ratio method can usually apply to both the stress and load factors because the intention at design loads is to maintain the elastic character of the structure. The stress factor, however, is usually a functional factor when used in the manner mentioned previously. Freudenthal[27] recommends elastic analysis and Sawyer[20] suggests the use of a functional load factor defined as the ratio of load to cause first yield: design load as a method to limit qualitatively deflections to steel structures designed by the rigid plastic method. This is equivalent of specifying a functional load factor in terms of a proof load previously mentioned. Generally, the ratio method for functional failure is applied to working loads alone, the more occasional loads being considered to produce little functional damage. Horne[13] has attempted to specify the wind loads with respect to the damage done to a structure by such loads in its lifetime. In this case the cumulative damage is considered in the specifications of design loads and does not enter the functional failure analysis with respect to factor of safety. The specification of functional failure safety level by the ratio method consists of specifying the ration of a value of the strength, S_F, associated with functional distress: working loads, L_1.

In the probability method, a value of probability of functional failure is specified which should not be exceeded in design. This probability is that of working loads exceeding the functional strength of the structure.

In the combined probability and ratio method, a minimum ratio is specified to ensure that a given probability of functional failure is not exceeded.

A. G. Pugsley[18] indicates examples from aircraft design involving functional safety. With aircraft structures of wood, failure was sudden, and the safety level was specified in terms of collapse alone. As metal structures were introduced it was found that structural distortion well below failing conditions could possibly be dangerous. To limit this, design regulations required specification of ultimate strength, and a relationship between ultimate and proof strength. This relationship was 0.625 in civil, and 0.75 in military aircraft. Also, in Fig. 6 (a) the various speeds, and accelerations are plotted for proof, and ultimate conditions. The flying limit is within the proof limit. The number

of occurrences of conditions beyond the proof limit can be found in Fig. 6 (b), where the bars of frequency of occurrence are shown.

Generally, the applicable discussion associated with the three methods and collapse apply for functional failure. The strength associated with functional failure is usually the load or the stress at which yield occurs in steel. In reinforced concrete the load or stress in the reinforcement associated with damaging cracks, or an acceptable amount of inelastic strain provides the criterion. In structures with a non-linear stress: strain response the stress or a load producing an acceptable amount of inelastic strain provides the criterion. In the cases where acceptable inelastic strain is the criterion, analysis by load factor methods is rationally employed.

The second method of providing a level of safety against functional failure involves specifying definite limits to deflection, vertical acceleration and other characteristics which, when exceeded may cause some restricted use of the structure. For instance, when plaster ceilings are used the deflection of beams is usually limited to 1/325 of the span to avoid plaster cracking. On other similar, but unplastered buildings, deflections greatly in excess may be permissible. In another instance, pedestrians, or people in stationary vehicles on bridges may suffer sickness and headaches because of structural vibrations. Limits suggested are for vertical acceleration of 0.5 f p s^2 by Holland,[7] and 0.1 g by Eastwood,[7] and the following:[22]

$$1 - 6 \text{ cps} \text{ - - - - - - } a f^3 = 2$$

$$6 - 20 \text{ cps} \text{ - - - - - } a f^2 = 0.33$$

in which a is the amplitude of structural vibration, in inches, and f equals the frequency of structural vibration in cps. These examples indicate the application of the second method of specifying a level of functional safety.

STRUCTURAL LIFE

The effects of fatigue, wear, and corrosion necessitates a definite life to a structure. Also, it has been illustrated that in aircraft the chances of failure in a number of hours of flying was used in specifying a safety level. With mobile structures, and structures supporting pulsating loads, the concept of limited structural life is not very difficult to appreciate. In the design stage of mobile structures the anticipated number of hours of use is an important consideration, and the attitude will vary with the time involved; for instance, greater care would be taken to provide for limited wear, deterioration, and fatigue effects on a vehicle intended for a few years of use, whereas, in the launching system for heavy precast concrete girders on a bridge, these points would be given little attention, large functional deformations may be allowed, and the ultimate level of safety may be reduced. In mobile structures the tendency is to consider the intended use, and the profitable working life in design.

In static structures the tendency is to design on a monumental basis. In earlier centuries, this was reasonable as the sufficiency of a structure did not vary from era to era. Today, especially with human rather than natural loads, the structures may be inadequate within ten years. If, however, the structure was designed on a monumental basis, with a high safety level, and therefore expensive, then it is likely that it will be retained, even if inadequate in use.

If this structure is allowed to exist over a long period of time it will be a monument, and revered because of its age. This attitude is not uncommon in the old world, where inelegant structures, inadequate in use, are often retained because of the money invested in them and their age.

This matter requires consideration in static structures under two headings. First, the safety level to be provided, and second, the loads to be designed for with human agency loadings. In the first case it is possible to consider the life of a structure with its cost. If it is necessary to consider the cost of inconvenience, death, and injury, associated with structures not built (Asplund[2]), then it would appear reasonable to consider the cost of inconvenience, injury, and death, associated with the possible non-replacement of a structure when it has become inadequate in the future. Thus, if the cost of this non-replacement was added to a structure it might be more economic to impose a lower level of safety, produce a cheaper structure, and replace it when inadequate. In design the safety level is usually provided to exist at the end of the structure's useful life. If the intended life is known, then the effects of deterioration, wear, and fatigue can be accurately allowed for in the selection of sections. If the intended life is not known, then larger sections, special details, and permanent materials must be provided for an indefinite life at greater expense. This is reasonable where monumental structures are involved, but not for purely functional structures. These points indicate the necessity of a decision on the anticipated life of a structure. This may be possible on a national basis; for instance, the intended life span of bridges constructed, and could be used in the selection of a level of safety, where the design is for a limited life. If, in fact, it is necessary to extend this in use, then a reasonable structural solution to the repair work necessary for this extra life span to be safe, can be made. Often a decision on the intended life span is impossible, and the design must be on a monumental basis.

The second heading may provide a reason to always design on a monumental basis. A case has been made[30] to legally prevent higher vehicle weights because of the large investment in bridges of a definite load capacity. This perpetuates the design loading used in the existing structures. In California 96% of the bridges are less than 100 ft long. The large majority of these bridges are designed for a H-20, A.A.S.H.O. vehicle loading, as are most of the longer bridges. The present design live loading is H 20- S 16. Under these circumstances, it is suggested that because of the heavy capital investment in H - 20 capacity bridges, it would be illogical to increase the design loading beyond its present level. If the design loading was increased it would mean more expensive structures, and the replacement of existing bridges, which would then be sub-standard, at the cost of hundreds of millions of dollars. Therefore, it is suggested that it would be reasonable to maintain the present design loading, and legally limit vehicle loads. This argument can be applied to most types of structures.

In the one case the argument is to design for a definite life span, and then replace the structure; in the other case to design on a monumental basis, and limit the loading. The problem may be solved by the consideration of the optimum measures to give the greatest public economic gain. In the case of bridges, this may result in the provision of routes of different load capacity.

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30 "Vehicle Loads and Highway Bridge Design," by S. Mitchell and G. F. Boorman, Proceedings, ASCE, Vol. 83, No. ST 4, July, 1957.

Journal of the

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

PERIODS OF FRAMED BUILDINGS FOR EARTHQUAKE ANALYSIS

By Mario G. Salvadori,[1] F. ASCE, and Ewald Heer[2]

SYNOPSIS

The shear, bending, rocking, and translational periods of cantilever beams with linearly varying shear and flexural rigidities, and with or without a concentrated mass at their top, are computed and combined to obtain a formula for the approximate evaluation of periods of framed buildings. This formula is shown to check the experimental value of the fundamental period of one well-known building within about 5% to 10% and to cover the complete range of periods corresponding to the actual range of parameter values encountered in practice. It is suggested that a more accurate formula, possibly based on the present results, could be adopted in the codes to improve the reliability of earthquake analysis.

INTRODUCTION

The maximum shear V_o at the base of a building due to an earthquake is a function of the earthquake acceleration and of the elastic properties of the building. M. A. Biot has shown[3,4,5] that a safe, approximate evaluation of V_o

Note.—Discussion open until May 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 12, December, 1960.

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3 "A Mechanical Analyser for the Prediction of Earthquake Stresses," by M. A. Biot, Bulletin of the Seismological Society of America, Vol. 31, No. 2, April, 1941.

4 "Characteristics of Strong Motion Earthquakes," by G. W. Housner, Bulletin of the Seismological Society of America, Vol. 37, No. 1, January, 1947.

5 "Spectrum Analysis of Strong Motion Earthquakes," by G. W. Housner and J. L. Alford, Bulletin of the Seismological Society of America, Vol. 43, No. 2, April, 1953.

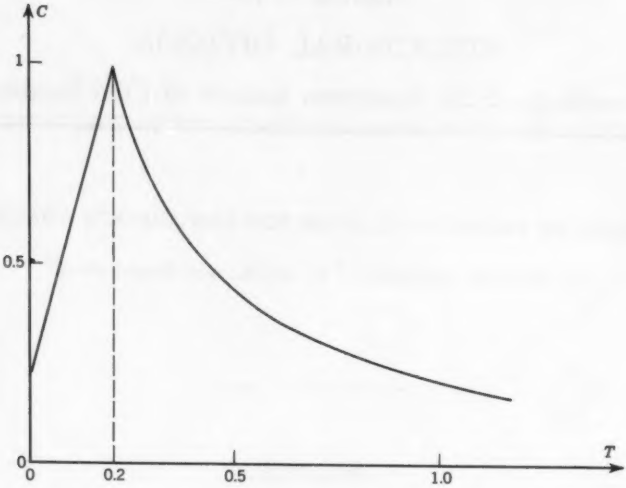

FIG. 1

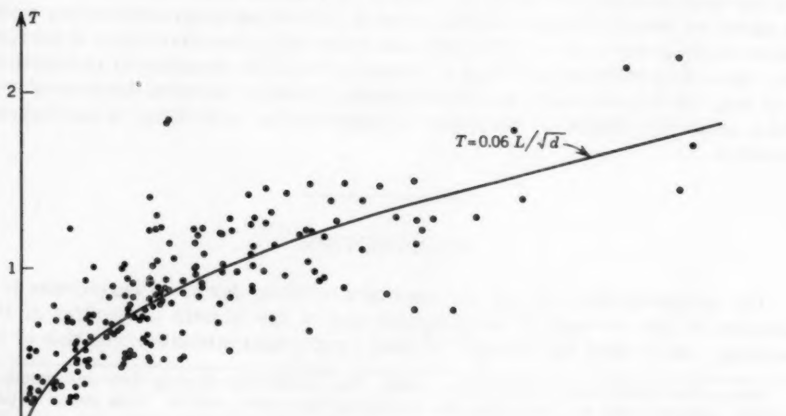

FIG. 2

can be obtained in terms of the weight W of the building by

$$V_o = C(T)\ W \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which C(T), the so-called response spectrum, is a function of the fundamental period T of the building. The spectrum has been evaluated with the help of analog computers on the basis of available earthquake records[3,5,6,7] (Fig. 1) and is widely used in earthquake analysis. Fig. 1 shows that an accurate evaluation of C requires a fairly precise knowledge of T. Fig. 2 indicates the scattering of the period T as a function of the dimensional parameter L^2/d (the height L and the width d of the building, in feet), and gives the graph of the period formula suggested elsewhere:[7]

$$T = 0.06 \frac{L}{\sqrt{d}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The error in Eq. 2 is shown in Fig. 2 to be as high as +100% and -50%. A simple formula is obtained subsequently that can be used to approximate the actual period of a building with greater accuracy, thus allowing a better evaluation of the maximum shear V_o.

SHEAR CANTILEVER WITH LINEARLY VARYING RIGIDITY

The periods of an n-story framed building due to shear deformations have been computed[8] and are often approximated by the periods of a shear beam with constant mass per unit of length m (kips sec^2/ft^2) and rigidity k (lb ft/ft). In actual buildings, m is practically constant up to the (n-1)-th story (while the n-th story is sometimes heavier), but k is found to vary from floor to floor. It will be assumed here that:

1. m is constant;
2. a concentrated mass M_o is located at the top of the building;
3. k varies linearly from a value k_o at the top of the second floor to a value $k_o(1-r_s)$ at the top of the building; and
4. $k = k_o/2$ at the top of the first floor, at which the shear resistance is usually reduced because of reduced partitions and a smaller number of columns.

With the symbols of Fig. 3, and noticing that in a shear beam the shear V is related to the slope by

$$V = k \frac{dy}{dz} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

the equation of motion of the beam becomes:

$$m \frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial z}\left(k \frac{\partial y}{\partial z}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

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6 "Aseismic Design of Structures by Rigidity Criterium," by E. Y. W. Tsui, ASCE Proceedings, February, 1959, Vol. 85, No. ST 2.

7 "Lateral Forces of Earthquakes and Wind," by the Joint Committee of San Francisco, ASCE, Transactions, Vol. 117, 1952.

8 "Natural Periods of Uniform Cantilever Beams," by L. S. Jacobsen, ASCE, Transactions, Vol. 104, 1939, p. 402.

Substituting in Eq. 4:

$$y = \phi(z) \cos p\,t \quad \dots\dots \quad (5)$$

and

$$k = k_o \left(1 - r_s \frac{z}{L'}\right) \quad \dots\dots \quad (6)$$

in which

$$L' = \frac{n-1}{n} L \quad \dots\dots \quad (7a)$$

and

$$x = \left(1 - r_s \frac{z}{L'}\right) \quad \dots\dots \quad (7b)$$

the differential equation for the mode ϕ corresponding to the frequency p becomes

$$\frac{d^2\phi}{dx^2} + \frac{1}{x}\frac{d\phi}{dx} + \left(p^2 \frac{m\,L'^2}{k_o\,r_s^2}\right)\frac{1}{x}\,\phi = 0 \quad \dots\dots \quad (8)$$

Its general solution is

$$\phi(x) = A\,J_0\left(a_s \sqrt{x}\right) + B\,Y_0\left(a_s \sqrt{x}\right) \quad \dots\dots \quad (9)$$

in which J_0 and Y_0 are Bessel functions of order zero, and

$$a_s = 2\,p\,\frac{L'}{r_s}\sqrt{\frac{m}{k_o}} \quad \dots\dots \quad (10)$$

Because both k and the total mass supported by a column at a level z (Fig. 3), are proportional to $\left(1 - r_s \frac{z}{L'}\right)$, the ratio of the rigidities at z = L' and z = 0,

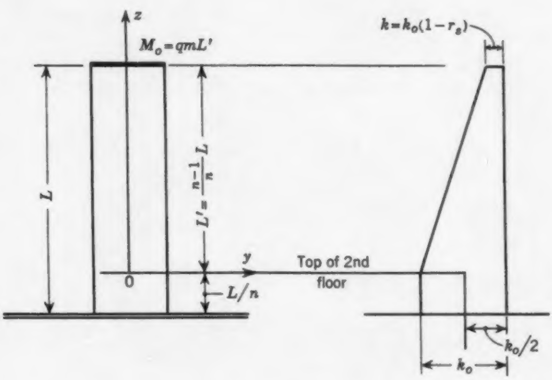

FIG. 3

and of the masses $M_o = q(m\,L')$ and $M_o + m\,L'$ at the same levels are equal and

$$r_s = \frac{1}{1+q} \quad \dots\dots \quad (11a)$$

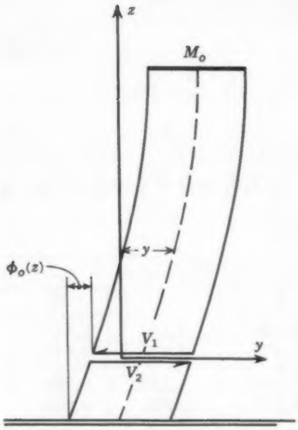

FIG. 4

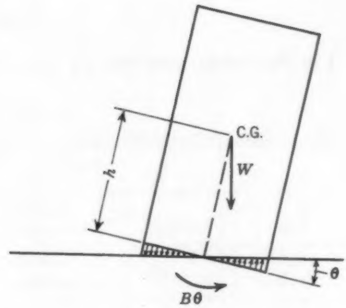

FIG. 5

and

$$q = \frac{1-r_s}{r_s} \quad \ldots\ldots\ldots\ldots\ldots (11b)$$

The boundary conditions associated with Eq. 8 state the equality of the shears:

1. V_2 at the top of the second floor and V_1 at the bottom of the portion of the building above the second floor (Fig. 4),

$$k_o \left.\frac{d\phi}{dz}\right]_{z=0} = \frac{k_o}{2} \left.\frac{\phi}{L/n}\right]_{z=0} \quad \ldots\ldots\ldots\ldots\ldots (12a)$$

or, with $L' = \frac{n-1}{n} L$ and by Eq. 7:

$$\phi\Big]_{x=1} = - \frac{2\, r_s}{n-1} \left.\frac{d\phi}{dx}\right]_{x=1} \quad \ldots\ldots\ldots\ldots\ldots (12b)$$

2. At the top of the n-th floor and at the bottom of M_o,

$$M_o \left.\frac{\partial^2 y}{\partial t^2}\right]_{z=L'} = k \left.\frac{\partial y}{\partial z}\right]_{z=L'} \quad \ldots\ldots\ldots\ldots\ldots (13a)$$

or, by Eqs. 5, 7, 10, 11:

$$\left.\frac{d\phi}{dx}\right]_{x=1-r_s} = \frac{a_s^2}{4}\, \phi\Big]_{x=1-r_s} \quad \ldots\ldots\ldots\ldots\ldots (13b)$$

By Eqs. 9, 12b, and 13b the frequency equation becomes

$$\begin{vmatrix} \left[J_1\left(a_s\sqrt{1-r_s}\right) + \frac{a_s}{2}\sqrt{1-r_s}\; J_0\left(a_s\sqrt{1-r_s}\right)\right] & \left[Y_1\left(a_s\sqrt{1-r_s}\right) + \frac{a_s}{2}\sqrt{1-r_s}\; Y_0\left(a_s\sqrt{1-r_s}\right)\right] \\ \left[J_0(a_s) - \frac{r_s}{n-1}\, a\, J_1(a_s)\right] & \left[Y_0(a_s) - \frac{r_s}{n-1}\, a\, Y_1(a_s)\right] \end{vmatrix} = 0 \quad \ldots (14)$$

Table 1 gives the coefficient α_{si} in the shear period formula

$$T_{si} = \alpha_{si}\sqrt{\frac{m\ L'^2}{k_o}} \qquad (15)$$

in which

$$\alpha_{si} = \frac{4\ \pi}{a_{si}\ r_s} \qquad (16)$$

i is the mode number, r_s has been chosen equal to 1.0. and 0.9 since M_o is

TABLE 1.—VALUES OF α_{si}

r_s	n	Value of i: 1	2	3	4	5
1.0	6	6.315	2.666	1.650	1.183	0.919
0.9		5.485	2.058	1.232	0.874	0.679
1.0	11	5.764	2.497	1.579	1.149	0.900
0.9		4.880	1.903	1.181	0.851	0.668
1.0	∞	5.225	2.277	1.452	1.066	0.842
0.9		4.263	1.688	1.063	0.781	0.619

seldom larger than 0.1 m L', and the number of stories n is chosen equal to 6, 11, or ∞.

BENDING CANTILEVER WITH LINEARLY VARYING RIGIDITY

The periods of cantilever beams with linearly varying flexural rigidity have been computed by W. Hort and A. Thoma[9] using Rayleigh's principle and the modes of the beam with constant flexural rigidity. Their procedure can be easily modified to take into account a concentrated mass $M_o = q'\ m\ L = q\ m\ L'$ at the top of the beam.

Given the moment of inertia

$$I = I_o\left(1 - r_b\frac{z}{L}\right) \qquad (17)$$

the frequencies p_i are expressed in terms of the frequencies p_{oi} for $r_b = 0$ by

$$p_i^2 = p_{oi}^2\left(1 - r_b\ s_i\right) \qquad (18)$$

in which

$$s_i = \frac{\int_0^1 x\ (\phi_i'')^2\ dx}{\int_0^1 (\phi_i'')^2\ dx} \qquad (19)$$

9 "Differentialgleichungen der Technik und Physik," by W. Hort and A. Thoma, Leipzig, 1939.

ϕ_i is the i-th mode of the beam with $I = I_o$, primes indicate differentiatión with respect to x, and p_{oi} is obtained interms of the roots a_{bi} of the transcendental equation:

$$\cos a_b \cosh a_b + 1 = \frac{n-1}{n} q\, a_b \left(\cosh a_b \sin a_b - \cos a_b \sinh a_b\right) \quad . . . \quad (20)$$

(in Eq. 20 $q = \frac{M_o}{m\ L'}$ is chosen in accordance with Eqs. 11).

TABLE 2.—VALUES OF α_{bi}

r_s	n	Value of i				
		1	2	3	4	5
1.0	6	1.787	0.285	0.102	0.0520	0.0314
0.9		2.095	0.322	0.113	0.0567	0.0338
1.0	1'	1.787	0.285	0.102	0.0520	0.0314
0.9		2.121	0.325	0.113	0.0567	0.0338
1.0	∞	1.787	0.285	0.102	0.0520	0.0314
0.9		2.151	0.328	0.114	0.0570	0.0340

Table 2 gives the coefficient α_{bi} in the bending period equation for $r_b = 0$:

$$T_{bi}\Big]_{r_b=0} = \alpha_{bi}\sqrt{\frac{m\ L^4}{E\ I_o}} \quad . . . . \quad (21)$$

in which

$$\alpha_{bi} = \frac{2\ \pi}{a_{bi}^2} \quad . . . . \quad (22)$$

and E is Young's modulus. The periods for $r_b \neq 0$ are given by

$$T_{bi} = \frac{\alpha_{bi}}{\sqrt{1-r_b\ s_i}} \sqrt{\frac{m\ L^4}{E\ I_o}} \quad . . . . \quad (23)$$

in which s_i is given in Table 3.

ROCKING MOTION

Biot[10] established the elastic stiffness coefficient for the rigid-body rocking motion

$$B = \frac{\pi}{16} \frac{E_s\ b\ d^2}{1 - \nu_s^2} \quad . . . . \quad (24)$$

10 "Analytical and Experimental Methods in Engineering Seismology," by M. A. Biot, ASCE, Transactions, Vol. 108, 1943, p. 365.

in which E_s and ν_s are Young's modulus and Poisson's ratio for the soil, respectively, and b and d are the horizontal dimensions of the rectangular build-

TABLE 3

i	1	2	3	4	5	∞
s_i	0.193	0.406	0.468	0.483	0.490	0.500

ing in directions perpendicular and parallel to the direction of the oscillations, respectively.

The rocking period is given by

$$T_r = 2\pi\sqrt{\frac{I_m}{B - W\,h}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

in which I_m is the moment of inertia of the total mass of the building about the center line of the foundation, W is the total weight of the building including the foundation, and h is the height of the building's centroid over the foundation (Fig. 5). The term W h is, in most cases, negligible compared to B.

TRANSLATIONAL MOTION

The period of the rigid body translational motion of the building imbedded in a soil, capable of reacting elastically in the horizontal direction, is given by

$$T_t = 2\pi\sqrt{\frac{W/g}{k_s\, b\, d_f}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (26)$$

in which k_s is the horizontal soil modulus, b is the width, and d_f is the depth of the foundation.

THE FUNDAMENTAL PERIOD OF A BUILDING

For buildings in which the torsional vibrations may be neglected, the fundamental period T may be evaluated approximately by assuming that the four types of motion considered above are practically uncoupled[11] so that

$$T^2 = T_s^2 + T_b^2 + T_r^2 + T_t^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (27)$$

(A more accurate, but much more complicated expression for T may be obtained by the procedure given elsewhere.[12])

Substituting in Eq. 27 the values of the periods T_1 given by Eqs. 15, 23, 25, and 26 we obtain:

$$T_1 = \sqrt{\alpha_{s1}^2 \frac{m\,L'^2}{k_o} + \frac{\alpha^2_{b1}}{1 - r_b\, s_1}\, \frac{m\,L^4}{E\,I_o} + 4\pi^2\left[\frac{I_m}{B - W\,h} + \frac{m\,L}{k_s\,b\,d_f}\right]} \quad \ldots (28a)$$

11 "Engineering Vibrations," by L. S. Jacobsen and R. S. Ayre, McGraw-Hill Book Co., Inc. New York, 1958.

12 "Earthquake Stresses in Shear Buildings," by M. G. Salvadori, ASCE, Transactions, Vol. 119, 1954, pp. 171-206.

An approximate evaluation of the periods of the higher modes may be obtained by dropping from Eq. 28a the rigid-body motions

$$T_i = \sqrt{\rho_{si}^2 \frac{m L'^2}{k_o} + \frac{\rho^2 b_i}{1 - r_b s_i} \frac{m L^4}{E I_o}} \qquad (i = 2,3,4,5) \ldots\ldots\ldots (28b)$$

In order to evaluate the accuracy of these formulas, the fundamental and the higher order periods obtained by Eqs. 28a and 28b, are compared in Table 4 with those determined experimentally by J. A. Blume[13] in a building in San Francisco, Calif., for which all the necessary parameters are given as follows:

M_o = 25 kips sec^2/ft

W = 16,000 kips

m = 2.2 kips sec^2/ft^2

n = 15

L = 196 ft

L' = 176 ft

h = 100 ft

q = 0.065

r_s = 0.940

E = 4,320,000 kips/ft^2

North-South direction

d = 69 ft

b = 60 ft

k_o = 3,410,000 kips ft/ft

I_o = 5,290 ft^4

r_b = 0.6

I_m = 7,650,000 kips ft/sec^2

B ≅ 2.84 x 10^9 kips ft

k_s b d_f ≅ 7,160,000 kips/ft

East-West direction

d = 60 ft

b = 69 ft

k_o = 3,890,000 kips ft/ft

I_o = 3,660 ft^4

r_b = 0.7

I_m = 7,600,000 kips ft/sec^2

B ≅ 2.14 x 10^9 kips ft

k_s b d_f ≅ 7,160,000 kips/ft

It is seen from Table 4 that the fundamental period predicted by Eq. 28a has an error of only 4.5% to 8.7%.

Table 5 compares the periods obtained by Eq. 15 with those measured on four different shear-stiffness analogs of a building.[14]

The accuracy of the formulas is seen to be satisfactory, to all practical purposes, in this case also, even though it is not easy to assess the accuracy of the analog results.

In order to assess the over-all accuracy of Eq. 28a, this may be simplified by: (a) setting L' = L and M_o = 0; (b) neglecting translation, and the term W h in comparison with B; (c) assuming that the building is a rectangular cross section beam, so that $I_m = m\,L\left(\frac{L^2}{3} + \frac{d^2}{12}\right)$. Factory the term $L/\sqrt{d}$, Eq. 28a

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13 "Period Determination and Other Earthquake Studies of a Fifteen-Story Building," by J. A. Blume, Proceedings, World Conference on Earthquake Engineering, Chapt. 11, June, 1956.

14 "Structural Dynamics in Earthquake-Resistant Design," by J. A. Blume, ASCE, Proceedings, Vol. 84, No. ST 4, July, 1958.

TABLE 4

Mode	Calculated Values of T, in seconds, for							Total T, in seconds		Measured T, in seconds		Remark
	Shear		Bending		Rocking		Translation					
	N-S	E-W	N-S	E-W	N-S	E-W	E-W N-S	N-S	E-W	N-S	E-W	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	0.732	0.684	0.803	0.974	0.327	0.375	0.053	1.14 7.5% to 10.5%	1.25 5.5% to 8.8%	1.23 1.25 1.27	1.32 1.33 1.37	Total
										(three independent measurements)		
2	0.294	0.275	0.135	0.167	. . .	. . .	. . .	0.324	0.322	0.38	0.42	Only shear and bending included
3	0.184	0.171	0.048	0.060	. . .	. . .	. . .	0.190	0.182	0.23	0.25	
4	0.133	0.124	0.025	0.031	. . .	. . .	. . .	0.135	0.128	0.167	0.186	
5	0.104	0.097	0.015	0.018	. . .	. . .	. . .	0.105	0.099	. . .	. . .	

TABLE 5

	k_o, in kips	Mode i	Periods calculated, in sec	Periods Measured on Analog System, in sec	% Difference
I	317000	1	2.42	2.68	9.7
		2	0.94	1.14	17.5
		3	0.585	. . .	. . .
		4	0.422	0.518	18.5
II	350000	1	2.30	2.64	13.0
		2	0.893	1.12	20.2
		3	0.555	0.703	21.0
		4	0.400	0.533	25.0
III	792000	1	1.53	1.94	21.1
		2	0.595	0.77	22.7
		3	0.370	0.494	25.0
		4	0.267	0.372	28.3
IV	233000	1	2.81	3.28	14.3
		2	1.10	1.35	18.5
		3	0.68	0.845	19.5
		4	0.491	0.642	23.5

becomes:

$$T_1 = \frac{L}{\sqrt{d}} \sqrt{\alpha^2_{s1} \frac{m_d}{k_o} + \frac{\alpha^2_{b1}}{(1-r_b\, s_i)} \frac{m\, d\, L^2}{E\, I_o} + 4\, \pi^2 \left[\frac{16\,(1-\nu_s^2)m}{E_s\, b\, d\, L}\left(\frac{L^2}{3} + \frac{d^2}{12}\right)\right]} \quad . \, . (29)$$

Eq. 29 would have the form of Eq. 2 if the square root at its right-hand member were constant. This is not so, and if the following practically extreme

variations are given to the parameters appearing in it:

$5.23 < \alpha_{s1} < 6.32$

$0.3 < r_b < 0.7$

$30 \text{ ft} < d < 150 \text{ ft}$

$50 \text{ ft} < L < 400 \text{ ft}$

$0.00312 \text{ kips sec}^2/\text{ft}^2 < m < 0.131 \text{ kips sec}^2/\text{ft}^2$

$1.64 \text{ ft}^4 < I_o < 1{,}060 \text{ ft}^4$

$12{,}900 \text{ kips ft/ft} < k_o < 155{,}000 \text{ kips ft/ft}$

$b = 1$

it is found that the graph of T versus L^2/d (Fig. 6) covers the complete scattering of experimental results shown in Fig. 2.

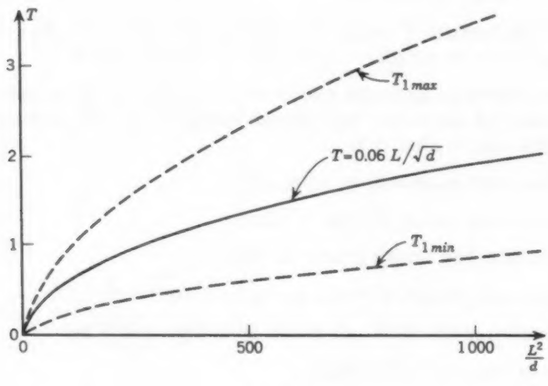

FIG. 6

CONCLUSIONS

It may be safely concluded that Eq. 29 is capable of predicting the fundamental period of rectangular buildings with an accuracy superior to that of Eq. 2. Hence, it is suggested that Eq. 29 may be taken as a basis for period evaluations in codes concerned with earthquake stresses in order to improve safety and accuracy in earthquake-resistant structures.

NOMENCLATURE

a = characteristic roots of frequency equations with subscripts s for shear, b for bending, i for mode;

b = width of building perpendicular to motion, in feet; as subscript refers to bending;

B = elastic stiffness coefficient for rocking on soil, in kips ft;

$C(T)$ = response spectrum as a function of building period T;

d = width of building parallel to motion, in feet;

d_f = depth of foundation below grade, in feet;

E = Young's modulus, in kips/ft^2;

E_s = Young's modulus for soil, in kips/ft^2;

g = acceleration of gravity, ft/sec^2;

h = distance from foundation bottom to center of gravity of building, in feet;

i = subscript indicating the mode;

I = moment of inertia of the area of a horizontal cross section of the building about its centroidal axis perpendicular to direction of motion, in ft^4;

I_o = I at the first story, in ft^4;

I_m = moment of inertia of mass of building about center line of foundation perpendicular to direction of motion in kips ft sec^2;

k = force required to displace relative to each other by a unit distance in the horizontal direction two cross sections, a unit distance apart, in shear motion, in kips ft ft;

k_s = horizontal soil modulus, in kips/ft^3;

k_o = k at the second floor, in kips ft/ft;

L = height of building above grade, in feet;

m = mass per unit height of building, in kips sec^2/ft^2;

M_o = concentrated mass at top of building in kips sec^2/ft;

n = number of stories in building;

p = angular frequency, in radians per second;

q = ratio of concentrated mass at top of building to total uniformly distributed mass above first floor;

r_s = variation of k per unit length;

r_b = variation of I per unit length;

t = time, in seconds;

T = period, in seconds;

V = shear at cross section z of building, in kips;

V_o = maximum shear at base of building, in kips;

W = total weight of building, in kips;

x = nondimensional height variable;

y = displacement, in feet;

z = dimensional height variable, in feet;

ϕ = mode; and

ν_s = Poisson's ratio of soil.

Journal of the
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

TENTATIVE RECOMMENDATIONS FOR THE DESIGN AND CONSTRUCTION OF COMPOSITE BEAMS AND GIRDERS FOR BUILDINGS

Progress Report of the Joint ASCE-ACI Committee on Composite Construction

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Note.—Discussion open until May 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 12, December, 1960.

INTRODUCTION

The Joint Committee was organized in 1956 to prepare recommendations for the design and construction of structures composed of prefabricated beams combined with cast-in-place slabs. After a review of the existing information and practices, the Committee has channeled one part of its activities toward preparation of recommendations for the design of composite beams and girders for buildings. The results of this work are reported herein. The Committee expects to prepare further reports after the completion of current studies.

The progress report is divided into two parts. The tentative design recommendations are presented in the first part. The second part contains explanations of the provisions of the design recommendations.

PART I. - TENTATIVE RECOMMENDATIONS

CHAPTER I. - GENERAL PROVISIONS

101 - Definition.

Composite beams and girders are comprised of prefabricated beams and of a cast-in-place reinforced concrete slab so interconnected that the component elements act together as a unit.

102 - Scope.

These recommendations apply to buildings subjected primarily to static loads. Structures containing prefabricated beams made of precast reinforced concrete, precast prestressed concrete, or either rolled or built-up steel sections are included. (Guides for the design of timber-concrete slabs and T-beams may be found in the technical literature (14).[1]

103 - Design of Slab.

The slab may be designed as either a one-way or two-way slab in accordance with the ACI Building Code (1). In the design of the slab, the stresses caused by composite action may be neglected.

104 - Design of Prefabricated Beams.

104.1 Precast Reinforced Concrete Beams.

Reinforced concrete beams should be designed according to the ACI Building Code (1) except as otherwise stated in Chapter 2 of these recomendations.

104.2 Precast Prestressed Concrete Beams.

Prestressed concrete beams should be designed according to the Tentative Recommendations for Prestressed Concrete (2) except as otherwise stated in Chapter 3 of these recommendations.

104.3 Steel Beams.

Steel beams should be designed according to the AISC Specifications (3), except as otherwise stated in Chapter 4 of these recommendations. (The use

[1] Numerals in parentheses—thus, (14)—refer to corresponding references in the Bibliography.

of plastic design for composite construction will be the subject for future recommendations of the Committee.)

105 - Effective Width of Flange

105.1 Flange on Both Sides of Beam.

For composite T-beams having the slab on both sides of the prefabricated beam, the effective width of the concrete flange should not exceed one fourth of the span length of the beam, and the overhanging width on either side of the prefabricated beam should not exceed eight times the thickness of the slab nor one half the clear distance to the next beam.

105.2 Flange on One Side of Beam.

For beams having the slab on one side only, the effective overhanging flange width should not exceed one twelfth of the span length of the beam, nor six times the slab thickness, nor one half the clear distance to the next beam.

106 - Mixed Construction.

The use of noncomposite beams in systems using composite beams is permissible provided that it is consistent with the deformational characteristics of the structure.

107 - Deflections.

107.1 Live-Load Deflections.

Live-load deflections should be calculated on the basis of the moment of inertia of the transformed composite section using the full value of the modulus of elasticity E_c.

107.2 Dead-Load Deflections.

1. For beams shored during construction, the dead-load deflections should be calculated on the basis of the moment of inertia of the transformed composite section using one half the value of the modulus of elasticity E_c.

2. For beams not shored during construction, the dead-load deflections should be calculated on the basis of the moment of inertia of the prefabricated beam alone except that deflections due to dead loads applied after the concrete slab has attained 75% of the specified 28-day strength should be calculated according to section 107.2.1.

108 - Continuity.

108.1 Determination of Moments, Shears, and Thrusts.

Moments, shears, and thrusts produced by external loads should be determined by elastic analysis. For the purposes of such analysis, the moment of inertia of the gross composite section may be used throughout the length of the beam.

108.2 Sections Resisting Negative Moments.

1. In negative-moment regions of continuous or cantilever beams, the bending moment may be assigned either to the prefabricated section alone or to the composite section composed of the prefabricated section and the slab reinforcement. Such assignment shall be consistent with the design of shear connection (section 108.3).

2. When the slab is continuous at the supports of beams, reinforcement should be provided sufficient to prevent excessive cracking of the slab.

108.3 Shear Connection in Regions of Negative Moments.

1. When the negative moments are assigned to the prefabricated section alone, shear connection between the prefabricated beam and the slab need not be provided in the regions of negative moments.

2. When the negative moments are assigned to the composite section, shear connection must be provided throughout the full length of the beam.

109 - Construction Methods.

109.1 Shoring.

When shores are used, they should be kept in place until the cast-in-place concrete has attained at least 75% of the specified 28-day strength.

109.2 Camber.

Necessary provisions should be taken in the design and construction to prevent excessive dishing of the slab in beams built with shores and excessive thickening of the slab in beams built without shores.

109.3 Treatment of Beam Surfaces.

Surfaces of prefabricated beams in contact with the slab should be cleaned of any foreign or loose material before casting the slab. It is preferable to leave the contact surfaces of steel beams unpainted.

CHAPTER 2. - SLAB ON PRECAST REINFORCED CONCRETE BEAMS

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201 - Allowable Stresses, Moduli of Elasticity, and Load Factors.

201.1 Allowable Stresses.

Allowable stresses for reinforcing steel and concrete specified in the ACI Building Code (1), are recommended for the design of composite beams. In structures composed of elements with different concrete strength, the allowable stresses in each portion should be governed by the concrete strength of the portion under consideration.

201.2 Tension in Concrete.

The tensile resistance of concrete should be neglected.

201.3 Moduli of Elasticity.

The following values for the moduli of elasticity should be used:

1. Steel E_s = 30,000,000 psi

2. Concrete E_c = 1,000 f'_c where f'_c is the 28-day compressive strength of the concrete under consideration.

201.4 Load Factors.

For designs based on section 203, load factors given in the Appendix to ACI Building Code are recommended.

202 - Determination of Flexural Stresses.

202.1 Design Method.

The design of reinforced concrete members may be made with reference to allowable stresses, working loads, and the accepted straight-line theory of flexure except in designs based on section 203.

202.2 Loading Conditions.

1. For unshored construction the dead load of the precast beam, and all other loads applied prior to the concrete slab attaining 75% of its specified 28-day strength should be assumed as carried by the precast beam alone. Live loads and dead loads applied after the concrete has attained 75% of its specified 28-day strength should be assumed as carried by the composite section.

2. For adequately shored construction all loads should be assumed as carried by the composite section.

202.3 Deformational Stresses.

Deformational stresses, including the effects of creep, shrinkage, and temperature, need not be considered except in unusual cases.

203 - Determination of Ultimate Flexural Strength.

203.1 Design Method.

Ultimate strength of a composite section may be calculated in the same manner as the ultimate strength of an integral member of the same shape following the procedure outlined in the Appendix to the ACI Building Code (1). In calculating the ultimate strength of a section, no distinction should be made between shored and unshored beams.

203.2 Limitations.

For beams designed on the basis of ultimate strength and built without shores, the effective depth of the composite section used in the calculation of the ultimate moment should not exceed:

$$d_c = \left(1.15 + 0.25\ \frac{M_L}{M_D}\right) d_p \qquad \text{. . . . . (1)}$$

in which d_c is the effective depth of the tension reinforcement in the composite section, M_L denotes the moment produced by live load and superimposed dead load, M_D is the moment produced by dead load prior to the concrete attaining 75% of the specified 28-day strength, and d_p represents the effective depth of the tension reinforcement in the precast section.

When the specified yield point of the tension reinforcement exceeds 40,000 psi, beams designed on the basis of ultimate strength should always be built with shores unless provisions are made to prevent excessive tensile cracking.

203.3 Construction Loads.

The precast beam alone should be investigated to assure that the loads applied before the concrete has attained 75% of its specified 28-day strength do not cause moment in excess of 1/1.8 times the ultimate moment capacity of the precast section.

204 - Determination of Shear.

204.1 Reinforcement of the Web.

Web reinforcement should be designed in the same manner as for an integral T-beam of the same shape. All stirrups should be extended into the cast-in-place slab.

204.2 Horizontal Shear.

1. The shear at any point along the contact surface may be calculated as:

$$v = \frac{V\,Q}{I} \qquad \qquad (2)$$

in which v is the horizontal shear per unit length of beam, V denotes the total external shear at the section considered caused by both live and dead loads, Q is the statical moment of the transformed area on one side of the contact surface about the neutral axis of the composite section, and I represents the moment of inertia of the transformed composite section neglecting the tensile resistance of concrete.

2. If the horizontal shear, v, exceeds the capacity of bond as recommended in section 205.3, shear keys should be provided throughout the length of the member. Keys should be proportioned according to the concrete strength of each component of the composite member.

205 - Shear Connection.

205.1 Shear Along Contact Surface.

Shear may be transferred along the contact surface by bond or shear keys. It should be assumed in the design that the entire shear is transferred either by bond or by shear keys.

205.2 Vertical Ties.

Mechanical anchorage in the form of vertical ties to prevent separation of the component elements in the direction normal to the contact surfaces should be provided. Spacing of such ties should not exceed four times the thickness of the slab nor 24 in. A minimum cross-sectional area of the ties in each foot of span of 0.15% of the contact area but not less than 0.20 sq in. is recommended. It is preferable to provide all ties in the form of extended stirrups.

205.3 Capacity of Bond.

1. The following values are recommended for the allowable bond stress at the contact surfaces:

a. When minimum steel tie requirements of section 205.2 are followed and the contact surface of the precast element is smooth (a smooth surface is one which has been cast against a form, trowelled, or floated). . . . . 40 psi

b. When minimum steel tie requirements of section 205.2 are followed and the contact surface on the precast element is rough . . . . 160 psi

c. When additional vertical ties are used the allowable bond stress on a rough surface may be increased at the rate of 75 psi for each additional area of steel ties equal to 1% of the contact area.

2. The capacity of bond at ultimate load may be taken as twice the values recommended in section 205.3.1.

CHAPTER 3. - SLAB ON PRECAST PRESTRESSED CONCRETE BEAMS

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301 - General.

Composite structures consisting of precast or cast-in-place slabs resting on prestressed concrete beams may be designed in accordance with the Tentative Recommendations for Prestressed Concrete (2), except that it is recommended to design the shear connection according to the provisions of sections 204 and 205 of these recommendations.

CHAPTER 4. - SLAB ON STEEL BEAMS

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401 - Allowable Stresses and Moduli of Elasticity.

401.1 Allowable Stresses.

1. The allowable stresses for steel, except for reinforcing bars, specified in the AISC Specifications (3) are recommended for the design of composite beams.

2. The allowable stresses for concrete and for reinforcing bars specified in the ACI Building Code (1) are recommended for the design of composite beams.

401.2 Tension in Concrete.

The tensile resistance of concrete should be neglected.

401.3 Moduli of Elasticity.

The following values of the moduli of elasticity should be used:

1. Steel: $E_s = 30,000,000$ psi.

2. Concrete: $E_c = 1,000\ f'_c$ where f'_c is the 28-day compressive strength of the concrete.

402 - Determination of Flexural Stresses.

402.1 Design Method.

1. Flexural stresses should be determined at the working-load level on the basis of the moment of inertia of the transformed composite section.

2. The transformed area of the composite section should be calculated on the basis of the modular ratio $n = E_s/E_c$.

3. For beams built without temporary supports and designed according to section 402.2, the section modulus of the composite section used in calculations should not exceed the value:

$$S_c = \left(1.35 + 0.35 \frac{M_L}{M_D}\right) S_s \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which S_c is the section modulus of the tension flange of the composite beam and S_s represents the section modulus of the tension flange of the steel beam alone.

4. The steel beam alone should be investigated to assure that the actual stresses in the steel do not exceed the allowable values for steel (section 401.1.1) before the concrete has attained 75% of its specified 28-day strength.

402.2 Loading Conditions.

It may be assumed in the stress calculations (except for provision of section 402.1.4) that all dead loads and live loads are resisted by the composite section whether or not temporary supports are used.

402.3 Deformational Stresses.

Deformational stresses, including the effects of creep, shrinkage, and temperature, need not be considered except in unusual cases.

403 - Shear.

403.1 Web Stresses.

The web of the steel beam should be capable of carrying the entire external shear without exceeding the allowable shearing stress.

403.2 Horizontal Shear Between Slab and Beam.

1. The horizontal shear at the junction of the steel beam and the concrete slab or haunch should be calculated as follows by Eq. 2 (section 204.2.1). In calculating the section properties, the modulus of elasticity of concrete E_c given by section 401.3.2 should be used.

2. The horizontal shear should be assumed to be transferred entirely by shear connectors except as noted in section 403.2.3.

3. In beams fully encased in concrete, the entire horizontal shear may be assumed to be transferred by bond and friction provided that such beams are designed in accordance with section 13 of the AISC Specifications (3). No shear connectors or ties are required in such beams.

403.3 Concrete Haunch.

The horizontal shearing stresses in the concrete haunch between the steel beam and the concrete slab should not exceed the allowable shear stress for concrete. If the allowable shear stress is exceeded, the haunch should be provided with web reinforcement. The shear connectors may be considered as web reinforcement.

404 - Shear Connectors and Ties.

404.1 Allowable Loads for Connectors.

The shear connectors should be designed on the basis of an allowable load given as follows:

1. Stud shear connectors

$$q = 165\, d_s^2 \sqrt{f_c'} \qquad (4a)$$

for h/d_s equal or larger than 4.2, and

$$q = 40\, h\, d_s \sqrt{f_c'} \qquad (4b)$$

for h/d_s smaller than 4.2, in which q is the allowable load per one stud, in pounds, h denotes the height of stud, in inches, d is the diameter of stud, in inches, and f_c' represents the 28-day compressive strength of concrete, in pounds per square inch.

2. Spiral shear connectors

$$q = 1900\, d_b \sqrt[4]{f_c'} \qquad (5)$$

in which q is the allowable load per one pitch of spiral, in pounds, and d_b denotes the diameter of bar, in inches.

3. Channel shear connectors

$$q = 90\,(h + 0.5\,t)\, w \sqrt{f_c'} \qquad (6)$$

in which q is the allowable load per one channel, in pounds, w represents the length of channel, in inches, t is the thickness of web, in inches, and h denotes the maximum thickness of flange, in inches.

4. For connectors other than the above, the allowable load should be developed from test data.

404.2 Spacing and Cover of Connectors.

1. Shear connectors should be spaced along the beam in accordance with the formula

$$s = \frac{q}{v} \qquad (7)$$

in which s is the spacing of shear connectors, q denotes the allowable load per shear connector or on a group of connectors placed at the same section, and v is the horizontal shear per unit length of beam defined in section 403.2.1.

2. Shear connectors should have at least 1 in. concrete cover in all directions.

404.3 Ties.

1. Mechanical ties should be provided between the slab and the beams to prevent separation. Such ties may be a part of the shear connectors.

2. The maximum spacing of ties should not exceed 24 in.

PART II. - EXPLANATIONS OF TENTATIVE RECOMMENDATIONS

The ASCE-ACI Joint Committee on Composite Construction has prepared these "Tentative Recommendations for the Design of Composite Beams and Girders in Buildings." Since no code is available for the design of composite buildings, the release of these tentative recommendations is considered desirable even though important research projects on composite beams are now in progress. As further data are developed from the research projects, the Committee expects to make appropriate modifications in its future reports.

In producing these recommendations, the Committee was faced with the problem of being consistent with existing codes for the materials involved. It felt that it was not the province of this Committee to rewrite accepted practice, but rather to correlate and draw on it-producing new concepts only where no accepted practice previously existed.

Lastly, the Committee felt that any recommendations produced because of the need for a useable guide should be written in terms familiar to most designers. Thus, the recommendations have been limited generally to working-stress design, even though several provisions are based on, or refer directly to, the conditions at ultimate strength.

The recommendations are limited to structures with concrete made from conventional hard rock aggregates. No data were available to the Committee on the behavior of composite beams made with lightweight aggregate concretes.

-2-

In order to explain the reasoning behind the Committee's recommendations, it will be necessary first to review the action of a simply-supported composite beam. After the prefabricated member has been erected, its lower flange is subjected to a tension which can be calculated by general flexural theories (Fig. 1). Immediately after casting, the slab which is still plastic adds no strength but merely dead load increasing the lower-flange stresses. After the slab has hardened, it becomes the top flange of the composite section; additional loads causing further tension in the lower flange are resisted by the entire composite section. However, this tension is less per unit load because the composite section is stronger. Since a unit load applies before commencement of composite action creates greater flange stresses than one applied after, attempts have been made to place as much of the load as possible on the composite section rather than on the prefabricated beam alone. The classic method was to shore the beam.

The discussion so far has dealt with stresses in the elastic range; that is, with stresses below the yield point rather than at the ultimate. The recommendations of the Joint Committee 323 (1), dealing with precast prestressed stems and cast-in-place slabs, state that the ultimate strength of the composite member is the same as that of a monolithic T-beam of equal dimensions. The fact that the composite beam may be built with or without shores does not influence the ultimate strength.

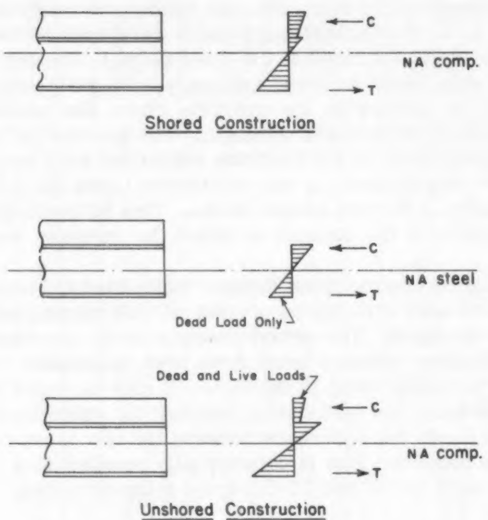

FIG. 1.—STRESS DISTRIBUTION AT WORKING LOAD

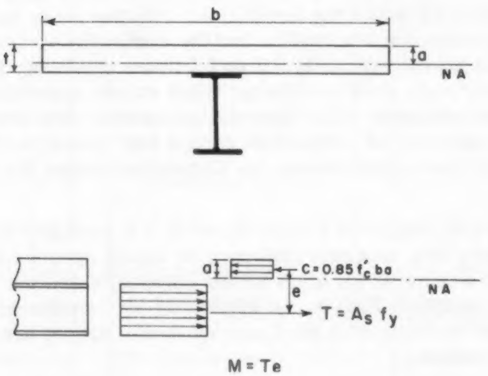

FIG. 2.—STRESS DISTRIBUTION AT ULTIMATE BENDING MOMENT

A similar situation exists in a steel beam with a cast-in-place slab. Assuming that at ultimate load the neutral axis lies in the concrete slab, the stress distribution corresponding to the ultimate bending moment may be approximated as shown in Fig. 2. (A similar theory can be developed for beams with the neutral axis below the top surface of the steel beam.) The entire tensile force is carried by the steel beam stressed uniformly to its yield point. The entire compressive force is carried by the concrete above the neutral axis uniformly stressed to 85% of its cylinder strength. The location of the neutral axis is such that the total force in the concrete equals the total force in the steel and the ultimate bending moment is this total force times the distance between the centers of gravity of the two stress blocks. This fully plastic stress distribution is independent of the manner in which the stresses are induced into the beam.

Table 1 contains comparisons between calculated and actual ultimate loads. Data for fifteen beams with depths of steel section varying between 3 in. and 24 in. have been tabulated. The actual ultimate loads are those observed in the tests. The calculate ultimate loads have been calculated from the reported properties of materials used in the tests. It may be noted that for all beams that failed in flexure, the correlation between the calculated and observed ultimate loads is good: the difference between the two loads varies between -4% and +11%. The observed load is substantially smaller than the calculated one only for specimens B21W and T3 designed with extremely weak shear connections.

The specimens listed in Table 1 are divided into two groups: those built without temporary supports and those either shored or loaded with additional weights during construction. Using the same method of calculation, the ultimate moment capacity was predicted with essentially equal accuracy for both shored and unshored beams. The ultimate flexural strength of a composite steel-concrete beam is independent of the construction method.

Based on a design stress of 20,000 psi and a yield stress of 33,000 psi, the factor of safety at first yielding of a non-composite steel beam is 1.65. At ultimate strength, the stress distribution is plastic and thus the factor of safety against failure is higher. For a symmetrical rolled section, the ultimate load is approximately 1.85 times the design load. On the other hand, for a composite beam designed on the assumption that the composite section takes all loads, the ultimate load is approximately 2.2 to 2.5 times the design load. The addition of a bottom cover plate does not change these values appreciably. If the composite beam were designed on the assumption that the steel beam alone resisted the dead loads, the ratio of ultimate to design load would be still higher.

As a result of these observations, the Committee makes the following recommendation:

> The design of composite beams made up of a prefabricated steel beam and a cast-in-place concrete slab may be based upon working stresses calculated by assigning all loads to the composite section regardless of construction method. That is, all loads may be considered in the design as resisted by the composite section even though shores are not employed during construction.

This recommendation is limited to composite sections with steel beams (section 402). It is not recommended for the design of composite sections with either precast reinforced concrete beams or precast prestressed concrete beams.

For sections composed of a slab and a precast reinforced concrete beam, the Committee recommends two alternate methods of design: a working-stress

TABLE 1.—COMPARISON OF [illegible]UTED ULTIMATE LOADS WITH TEST DATA

Specimen	Refer-ence	Depth of steel beam, in inches[a]	Average yield point, in ksi[b]	Size of slab, in in. by in.[a]	[illegible] e,	Type of shear connector	Ultimate Load[c]			Mode of failure
							Computed, in kips	Test, in kips	Col. 9 / Col. 8	
(1)	(2)	(3)	(4)	(5)	([illegible]	(7)	(8)	(9)	(10)	(11)
Beams without shoring[d]										
Double T	(2)	18	37.9	131x6	3.93	stud	156.3	172.8	1.11	flexure
C	(3)	8	37.6	24x4.9	3.60	bar + loop	54.5	57.2	1.05	flexure
B24W	(4)	24	36.8	72x6.3	5.10	channel	116.9	111	0.95	did not fail
B21W	(4)	21	37.0	72x6.1	5.75	channel	96.0	79	0.82	horizontal shear
T1	(5)	3	37.4	18x1.8	5.10	channel	13.0	13.0	1.00	flexure
T2	(5)	3	37.4	18x1.7	1.82	channel	9.7[e]	10.2	1.05	flexure
T3	(5)	3	37.4	18x1.8	4.25	channel	12.2	8.7	0.71	horizontal shear
1	(6)	8	43.8	30x4.5	7.04	spiral	75.7	77.0	1.02	flexure
2	(6)	8	39.0	30x4.5	7.38	spiral	65.0	70.5	1.08	flexure
Beams with shoring or preloaded[f]										
D	(3)	8	37.9	24x4.9	3.30	bar + loop	45.8	44.1	0.96	flexure
E	(3)	8	38.0	24x4.8	3.61	bar + loop	54.5	54.9	1.01	flexure
B24S	(4)	24	36.1	72x6.2	5.67	channel	115.1	115	1.00	did not fail
B21S	(4)	21	37.6	72x6.3	6.53	channel	99.2	102	1.03	flexure
3	(6)	8	39.8	30x4.5	7.04	spiral	66.0	67.5	1.02	flexure
4	(6)	8	42.7	30x4.5	7.38	spiral	74.5	71.8	0.96	flexure

[a] Rounded-off values. [b] Average weighted according to areas and corresponding yield points. [c] Dead load not included. [d] Average test/comp. for flexural failures: 1.052. [e] Neutral axis in the steel section; for all other specimens neutral axis in the slab. [f] Average test/comp. for flexural failures: 0.996.

design (section 202) and an ultimate-strength design (section 203). The working-stress procedure takes into account the construction method, while the ultimate-strength procedure is independent of the method of construction. Both procedures follow principles well established in the design of reinforced concrete.

The ultimate strength of a composite concrete T-beam is independent of the method of construction. Thus, the load factors used in the ultimate-strength-design provide the necessary overload capacity regardless of the type of construction used. The recommendation of the ultimate-strength design as an alternate method for reinforced concrete sections is warranted both by the availability of information and by the familiarity of the designers with such design.

The need for a working-stress procedure is self-evident. However, in contrast to composite sections with steel beams, the reserve strength of a composite concrete beam beyond first yielding of the reinforcement can be small. Therefore, it is recommended that the construction method in the working-stress design of concrete beams be considered.

Finally, the Committee considered satisfactory the currently available method for the design of composite sections made with prestressed concrete beams (Chapter 3).

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Although the procedure recommended for the design of sections with steel beams provides adequate safety against failure of the composite beam, it does not limit either the erection stresses or the actual stresses at working load. To guard against possible damage to the steel section, sections 402.1.3 and 402.1.4 place limits on the total stress and on the stresses in the steel beam prior to hardening of the slab.

In beams built with temporary supports, the actual stresses exceed the calculated values only as a result of volume changes of concrete. The increases in the governing tension in the steel section resulting from volume changes are usually small and may be safely neglected. On the other hand, in beams built without temporary supports, the actual stress may be substantially in excess of the calculated value because the dead load is in reality resisted by the steel section alone, while the recommended procedure assigns it to the composite section. The limit on the section modulus of the composite beam (section 402.1.3) places a maximum of 27,000 psi on the total stress caused by dead and live loads in an unshored beam. This provides safety against yielding comparable to that of a noncomposite beam.

To avoid damage to the steel section during construction, it is recommended in section 402.1.4 to limit the stresses in the bare steel beam before hardening of the slab to the conventional allowable values for steel. This provision is particularly needed for unsymmetrical steel sections with light compression flanges.

The ultimate-strength procedure for the design of sections with precast reinforced concrete beams provides adequate safety against failure of the composite beam, but additional limitations are needed to guard against damage to the precast beam during erection and against excessive widening of the tension cracks at working loads. The provisions of sections 203.2 and 203.3 serve this purpose.

Except for small changes caused by differential shrinkage, the stresses in a composite beam fully shored during construction are the same as in a cast-in-place monolithic beam. In beams built without temporary supports, the weight of the precast beam and of the concrete slab is resisted by the precast section alone. However, the ultimate strength of the composite section is the same

regardless of the construction procedure. Thus actual stresses caused by dead and live loads are higher if shores are not used. The limitation on the effective depth of the composite beam (section 203.2) limits the total stress caused by dead and live loads to 75% of the specified yield point f_y of the tension reinforcement.

The limit of 0.75 f_y not only keeps the stresses well below the yield point but also prevents excessive tensile cracking of the beam provided that the yield point is only moderately high. The Committee recommends that, unless provisions are made to prevent excessive widening of tensile cracks, composite beams designed on the basis of ultimate strength with the yield point of the steel in excess of 40,000 psi should not be constructed without the use of shores. When the yield point does not exceed 40,000 psi, the actual stresses in the tension reinforcement in a composite beam built without temporary supports will be no higher than the maximum values attainable in monolithic beams designed by the ultimate strength design procedure recommended in the ACI Building Code (1).

The provision of section 203.3 guards against damage to the precast section during construction.

-4-

It has been assumed tacitly in the preceding discussion that the shear connection between the prefabricated beam and the cast-in-place slab is capable of developing the ultimate moment of the composite section. It may be observed in Fig. 2 that at ultimate load, the shear connection must develop the fully plastic force C (or T) through horizontal shear in the length between the sections of zero and maximum bending moments. If the connection is able to yield and still develop the fully plastic or ultimate strength of the beam, the exact distribution of horizontal shear is not critical. On the other hand, if the connection is brittle or if its yielding causes a significant loss of composite action, the distribution of this horizontal shearing force has to be considered. Experimental studies are now in progress which are expected to supply the information needed for formulating an ultimate strength procedure for the design of the shear connection.

Another approach to the design of the shear connection makes use of the elastic horizontal shear formula. Such approach, based on experimental observations, was developed for the design of composite bridges (7) and was adapted for this report. It provides a connection capable of developing the fully plastic moment capacity of the composite beam.

The AASHO procedure (7), derived for steel-concrete beams, makes use of the useful capacity of mechanical connectors and of a variable factor of safety. The factor of safety is intended to furnish a connection capable of developing the ultimate flexural strength of the composite beam; it accounts for moving loads, for the different proportions of moments and shears resisted by the noncomposite and the composite sections, for the properties of the cross section and for the level of design stresses. The connection and the beam have to resist the same ultimate load. At working load, however, the beam is always resisting both the dead load and the live load while the connection may resist only the live load. In such case, the factor of safety needed in the design of the connection is higher than that of the beam. If, however, the live and dead loads are distributed in the same manner along the beam, as is usually the case in building design, and if the assumption is made that all loads are resisted by the composite section, then the required factor of safety for shear connectors is equal to the factor of safety of the beam. The allowable values for various types of con-

nectors may then be determined as the ratio of their strength to this constant factor of safety.

In the interest of simplicity, the Committee recommends that the horizontal shear be calculated from the elastic formula, using the vertical shear caused by all loads acting on the beam regardless whether shoring is used or omitted during construction (sections 204.2.1 and 403.2.1). It is important to note that the use of the total load in calculating the horizontal shear is one of the basic assumptions in the derivation of the recommended design procedure.

Composite sections with steel beams designed as recommended by this Committee usually have a factor of safety between 2.2 and 2.5. The useful strength of mechanical connectors, used on steel beams, may be determined from empirical formulas given in the AASHO specifications (7). In using the AASHO formulas, the Committee recognized their conservative nature in relation to static loading and, therefore, selected the safety factor of 2.0 as adequate when arriving at the recommended formulas for allowable loads (section 404.1). (Preliminary data from an investigation now (1960) in progress at Lehigh University, Bethlehem, Pa., have indicated that ultimate strength of a composite beam might be developed with a substantially smaller number of connectors than that obtained by the procedure recommended at present.)

The question of bond between the steel section and the concrete slab has always been a perplexing one. Tests have shown that it usually exists, but no one can guarantee its duration. For this reason, the Tentative Recommendations give no credit to bond. Fully embedded beams constitute the only exception to this rule (section 403.2.3).

For the transfer of shear between a concrete beam and a concrete slab, the use of bond or of shear keys is recommended. Several experimental studies and experience have shown that bond in conjunction with steel ties provides a very effective and reliable connection. However, the tests have shown also that there is a practical limit to the capacity of the bond connection.

In the 78 tests of composite beams studied by the Committee, nine beams failed in horizontal shear. The horizontal shear at which failure occurred (calculated from the elastic formula), the type of surface, and a description of steel ties are given in Table 2 for all nine beams. The six specimens with a smooth surface failed at horizontal shearing stresses ranging from 78 psi to 350 psi, while the three specimens with a rough surface failed at horizontal shearing stresses between 418 psi and 580 psi. The remaining 69 specimens, which did not fail in horizontal shear, included both rough and smooth surfaces. The maximum horizontal shears obtained in the tests of all 78 beams are shown in Fig. 3.

The data for specimens with smooth contact surface cover well the range of moderate to large slendernesses (ratios of shear span a to effective depth of tension reinforcement d ranging from 3 to 9.75). It may be noted in Fig. 3 and in Table 2 that the data representing horizontal shear failures suggest a possible decrease of strength with increasing slenderness. The data for specimens with rough contact surface are limited to the range of small and moderate slenderness (a/d ranging from 1 to 5); only two specimens with rough surfaces were slender (a/d = 7.25) and these did not fail. Furthermore, all three specimens with rough surface that failed in horizontal shear had (a/d) - ratios less than 4.25. It is hoped that an investigation now in progress will provide information on slender beams with rough surfaces.

In view of the evidence discussed briefly in the preceding paragraph, the Committee considered it advisable at this time to base the recommended bond

values for smooth surfaces on the strength of 80 psi, and for rough surfaces on the strength of 400 psi. The factor of safety for beams designed according to these recommendations ranges upward from 2.0. In view of the uncertainty concerning the bond strength in the range of larger (a/d) - values, a factor of safety of 2.5 was used to obtain the allowable bond stresses (section 205.3.1) for the rough surfaces. For smooth surfaces, better covered by the available test information, a factor of safety of 2.0 was selected.

The Committee strongly recommends the use of steel ties crossing the contact area in all composite concrete T-beams. For light concrete joists and for slabs with precast beams completely embedded on three sides, this recommendation may be too severe. The Committee considered such construction outside the scope of the Tentative Recommendations.

The minimum amount of ties recommended by the Committee is based on the value suggested by Committee 323 (2). A small increase is suggested in

TABLE 2.—HORIZONTAL SHEAR FAILURES OF COMPOSITE CONCRETE-CONCRETE BEAMS

Specimen (1)	Reference (2)	Type of joint surface (3)	Ties (4)	a/d[a] (5)	Horizontal shearing strength, in psi (6)
BS-I	(8)	Smooth	#3-6"o.c.	3.0	350
BS-II	(8)	Smooth	#3-16"o.c.	4.2	340
A2	(9)	Smooth	none	6.5	78
C2	(9)	Smooth	#4-6"o.c.	6.5	100
A3	(9)	Smooth	none	3.3	119
J	(10)	Smooth	none	8.0	122
BRS-I	(8)	Rough	#3-6"o.c.	3.0	450
BRS-II	(8)	Rough	#3-16"o.c.	4.2	580
III-0.6-1.66	(11)	Rough	#3-6"o.c.	3.8	418

[a] Ratio of length of shear span to effective depth of tension reinforcement.

bond values for beams with ties in excess of the minimum. This latter allowance is based on the results of recent tests of push-off specimens (8).

In view of a lack of experimental data, the Committee is not prepared to make any detailed recommendations concerning the design of shear keys.

-5-

As a general rule, composite beams which are shallower and used on longer spans than conventional beams are more deflection sensitive. The use of shores reduces the total deflection but creates construction problems. If the shore is properly placed, the deflection is a function of the load, span, and stiffness of the composite section. If the shore is driven too tight, the energy stored in the prefabricated beam will produce an additional effective load which will increase the deflection of the slab. Furthermore, if the floor is screeded level and the beam is shored high, the slab will be made too thin at midspan with resulting loss of strength.

In the case of an unshored beam, the deflection must be calculated separately for the dead load, resisted by the steel beam alone, and for the live load, resisted by the composite section. The total deflection is greater than for a beam built with shores, but the floor may be screened level at its final elevation. The beam itself must be checked for deflection under its weight plus that of the wet concrete and formwork to assure that excessive thickening of the slab does not occur at midspan. The Tentative Recommendations carry warnings about these potential problems of deflection.

-6-

The question of deformational stresses was investigated in some detail. Such stresses are always present in composite members at working load stress levels.

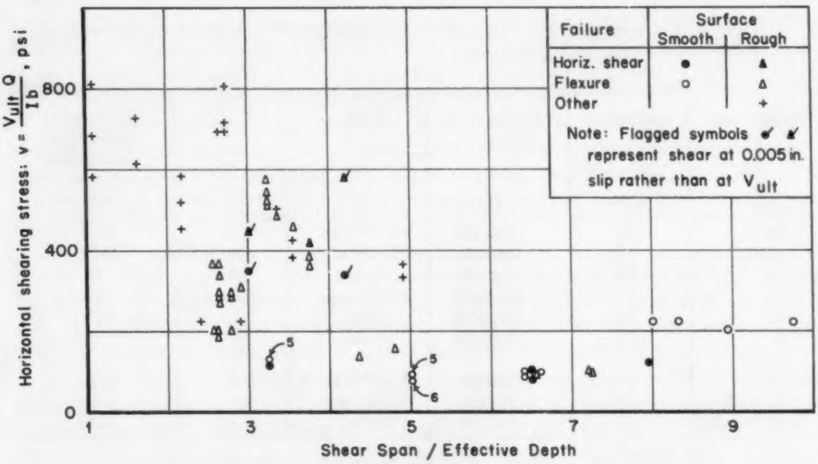

FIG. 3.—HORIZONTAL SHEAR IN CONCRETE-CONCRETE BEAMS

However, they are wholly internally balanced and hence have no effect upon the ultimate capacity. The Committee felt that in ordinary buildings deformational stresses may be safety ignored except in unusual cases.

Creep of concrete affects the deflections of a composite beam. In cases such as beams carrying heavy masonry partitions, the long-time effect could cause serious cracking of the partitions even though the strength of the beam would be unaffected. The simplest manner of correcting the deflection calculation for this long-time effect is to reduce the value of the modulus of elasticity of the concrete. It is recommended to use one half the short-time value of the concrete modulus to account for creep deflections.

The Tentative Recommendations contain a few guide lines for continuous design. The recommendations concerning the elastic properties of composite beams for the purpose of frame analysis follow the current practice for reinforced concrete T-beams. On the other hand, the recommendations for the design of the negative moment sections are in accord with the current bridge practice.

Finally, the recommendations concerning the effective slab width are based on the current practices both for composite beams and for reinforced concrete T-beams.

-7-

The recommendations have been prepared as a result of a need for a guide. Composite construction is being used more and more as its advantages are being demonstrated.

The task of the Committee is by no means finished. Its further work and direction of its investigations will be influenced by the reaction of the engineering profession to these Tentative Recommendations.

ACKNOWLEDGMENTS

The direction and coordination of the efforts leading to this progress report were assigned to Philip P. Page, Jr. The Committee wishes to acknowledge his efforts, which were primarily responsible for the prompt completion of the report.

Respectfully submitted,

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Journal of the
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

STRUCTURAL MODEL ANALYSIS BY MEANS OF MOIRÉ FRINGES

By A. J. Durelli[1] and I. M. Daniel,[2] A.M. ASCE

SYNOPSIS

The objective of this paper is to demonstrate the use of moiré fringes in the measurement of displacements and rotations in structural models. The method, applied to the case of a simply supported beam, a continuous beam, and two plane frames, gave results in satisfactory agreement with theory, whenever such comparison was made. The techniques used are extremely simple and inexpensive.

INTRODUCTION

The experimental analysis of structures has been approached by many different methods. In most cases a model geometrically similar to the prototype is built. The general approach consists in applying on the model loads similarly located and of the same relative magnitude as those of the prototype and in measuring and evaluating the results.

These results can be in the form of deflections measured by mechanical or optical means, or in the form of strains obtained by strain gages, or stresses determined photoelastically. The location of inflexion points evidenced by photoelasticity would also suffice for the analysis of the structure. Another direct method of experimental analysis is based on the slope deflection equations. It applies mainly to structures composed of prismatic bars and consists of measuring displacements and rotations at the ends of the members of the structure. The most widely applied method, however, is the

Note.—Discussion open until May 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 12, December, 1960.

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one based on the Maxwell-Betti reciprocal theorem. Influence lines for certain stress functions are obtained by applying rotations and displacements characteristic of the stress functions studies. The deflection curve of the structure for a unit displacement or rotation represents the influence line of the redundant characterized by this displacement or rotation. In connection with this approach, the Beggs deformeter has been widely used as a sensitive apparatus for the application of small displacements. The induced, equally small, displacements in the structure are measured by micrometric microscopes. This apparatus was used because of the requirement of the Theory of Elasticity that displacements and deformations be small. It was recognized for a long time that large displacements in structural models produce inadmissible errors often in excess of 10%. The first successful demonstration of the applicability of the large displacement method was made by W. J. Eney.[3] He showed experimentally that, if equal and opposite displacements or rotations are applied at one point of the model and if the algebraic differences of deflections in the structure are measured, the errors due to changes in geometry are eliminated. This was proved and stated in the form of a general theorem by C. Massonnet.[4] Thus, the use of finite displacements is perfectly valid and yields sufficiently accurate results, provided the displacements are accurately measured.

This paper deals with the application of moiré fringe patterns in the determination of these displacements. The moiré method is an accurate and practical method for measuring displacements of the order encountered in common structural models.

PROCEDURE

The moiré effect is an optical phenomenon observed when two arrays of lines are superimposed. When one such array is superimposed over another, a pattern of alternating dark and light fringes appears (Fig. 1). This is due to mechanical interference or shadow effect. The dark fringes appear at the points where the dark lines of one array cover the light spaces of the other. In the usual applications of moiré arrays of equidistant, parallel lines are used. When two superimposed sets of lines are matched, no fringes appear. When one set is rotated to make a small angle with the other, a set of parallel equidistant fringes perpendicular to the bissector of the angle appears (Fig. 1). The first dark fringe appears where a point of one set moves a distance equal to half the spacing of the array, or pitch, perpendicularly to the lines of the fixed array. Every light fringe of the moiré pattern is a locus of points of displacement equal to an integral multiple of the pitch in the direction perpendicular to the fixed set of lines. These fringes are ordered according to this integral multiple of the pitch to which they correspond. Moiré fringes are not only a result of rigid body translations and rotations, but may also be a result of deformation. When the movable grid of lines is on a plane surface of a deformable body, it follows the deformations of that body and superimposed on the fixed grid it produces fringes. These fringes are a result of

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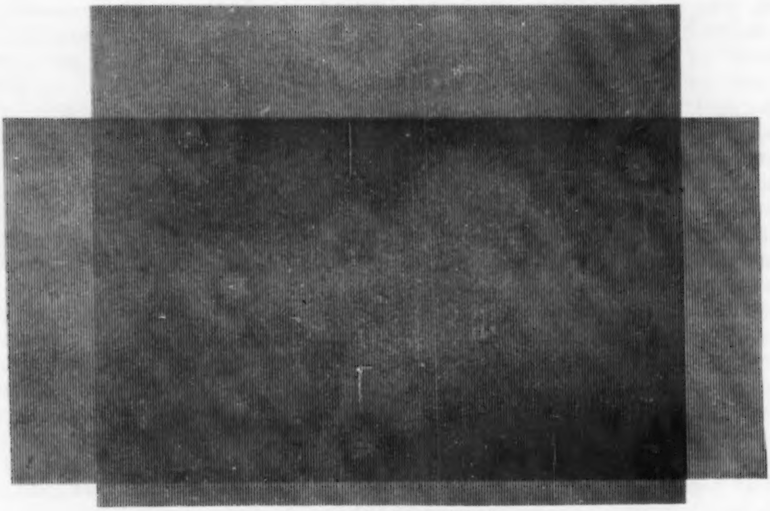
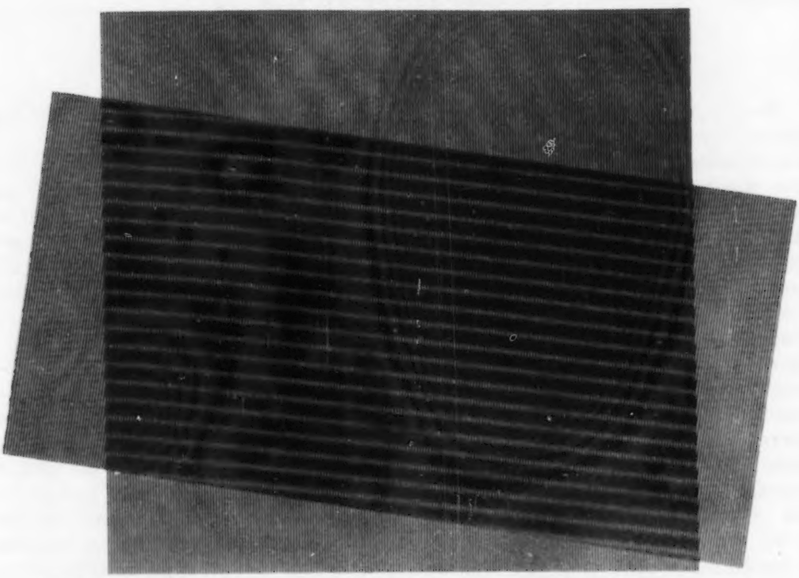


(a) MATCHED GRIDS



(b) ONE GRID DISPLACED WITH RESPECT TO THE OTHER. FRINGES ARE LOCI OF EQUAL HORIZONTAL DISPLACEMENT RELATIVE TO POSITION OF MATCHED GRIDS

FIG. 1.—MECHANISM OF FORMATION OF MOIRÉ FRINGES (GRIDS OF 60 LINES PER INCH)

rotations, displacements, and change of pitch (due to strain) in the deformed grid. The study of deformations by means of moiré fringes was presented by P. Dantu.[5] In stress analysis applications, one is interested in separating the effects of translations and rotations from those of strain. Some geometric properties of the moiré fringes and their application to strain analysis were described by S. Morse, A. J. Durelli, and C. Sciammarella.[6] In structural analysis, however, one is interested in measuring displacements of the centerline of members of structural models. To avoid the effects of strain, relatively coarse grids are used. One grid, referred to as model grid, is cemented to the model and the displacements are measured with respect to a fixed grid, referred to as reference or master grid. The moiré fringes are counted starting from a point of known displacement, preferably zero displacement.

Structural models of beams and plane frames were machined out of plexiglas. These models were mounted on plexiglas plates by means of screws through the points of support of the structures. Two ways of applying the grids were tried here. In one case, transparent sheets bearing prints of equidistant parallel lines were cemented to the models and plexiglas plates. These sheets are commercially available as Artype or Zip-a-tone sheets and used by artists and draftsmen. Grids of 50 and 60 lines per in. are readily available in this form. In order to increase the accuracy, a denser grid was needed. This was achieved by printing grids of 133 and 300 lines per in. on plexiglas. Structural models were machined from such printed plates.

RESULTS

Simply Supported Beam Under Pure Bending.—A simple application of the moiré method was made in the determination of the deflection line of a simply supported beam under pure bending. A beam 1 in. deep and 12 in. long was machined from a 1/4 in. thick plexiglas plate with a grid of 300 lines per in. printed on it. The grid lines were parallel to the axis of the beam. A similarly printed plate was held in contact with the beam such that no fringes appeared at zero load. When the load was applied, the model grid moved with respect to the fixed in space master grid and the fringe pattern shown in Fig. 2 appeared. The fringe orders start from zero at the supports and increase towards the center. One fringe order corresponds to 1/300 in. of displacement. A modulus of elasticity of 360,000 psi was determined from the maximum deflection at the center. Then, the theoretical deflection curve was computed and plotted in Fig. 2. Nearly all the experimental points fall on the curve. These points were determined from the moiré fringe orders along the centerline of the beam. For small deflections and coarse grids, the moiré fringes would be straight and nearly vertical. In the present case, the shape and inclination of the fringes, especially near the center of the beam, is due to the curvature of the model grid lines, to vertical strains in the beam, and to the fact that the model is not viewed exactly in a direction perpendicular to the plane of the grids. The curvature of the beam tends to tilt the fringes by

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6 "Use of Some Geometric Properties of Moiré Fringes in the Analysis of Strains," by S. Morse, A. J. Durelli and C. Sciammarella, presented at the October, 1960, ASCE Convention, Boston, Mass.

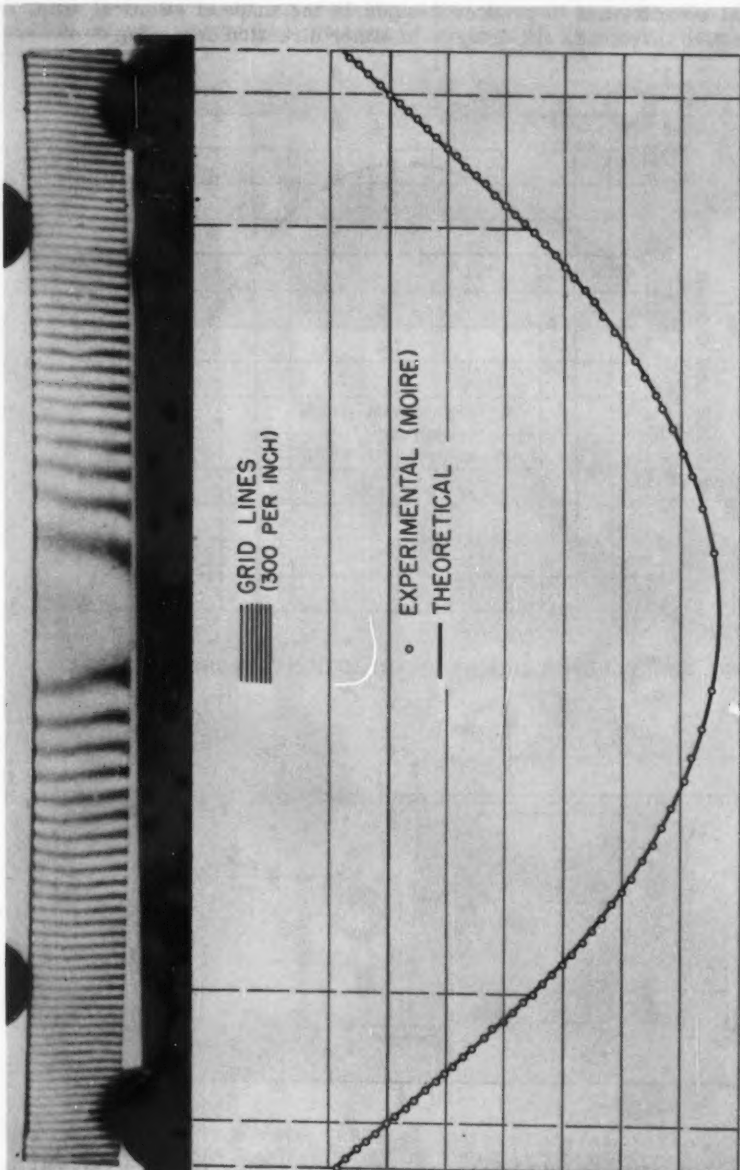

FIG. 2.—DEFLECTION LINE OF SIMPLY SUPPORTED BEAM UNDER PURE BENDING

making them perpendicular to the bissector of the angle of rotation. The vertical strains tend to produce fringes in the shape of elliptical arcs. The perspective effect can tilt fringes in either direction depending on the angle

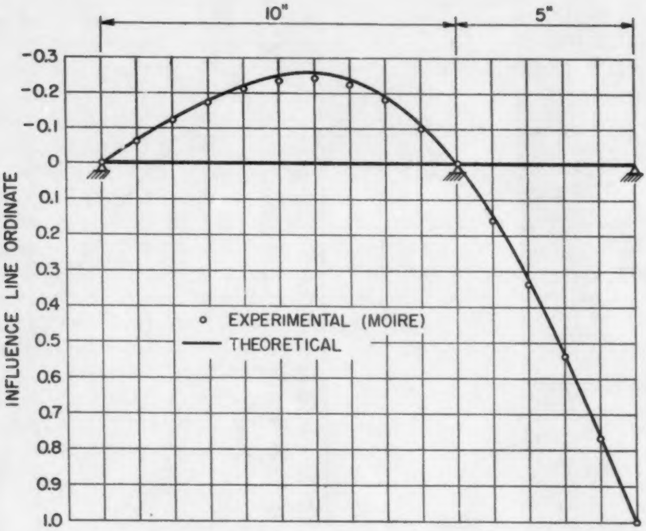

FIG. 3.—INFLUENCE LINE OF END REACTION OF A TWO-SPAN BEAM

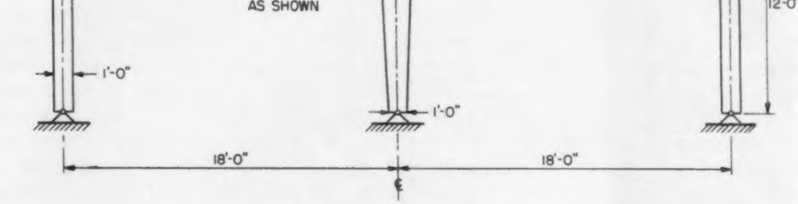

FIG. 4.—GEOMETRY OF THE FRAME ANALYZED BY MOIRÉ FRINGES AND THEORY

of view. All these factors, however, do not seem to influence the fringe order along the neutral axis of the beam as evidenced by the good agreement between theory and experiment.

Two-Span Continuous Beam.—A two-span continuous beam with spans 10 in. and 5 in. long was made out of plexiglas and screwed to a plexiglas plate at the points of supports (hinges). Horizontal grids of 52 lines per in. were used

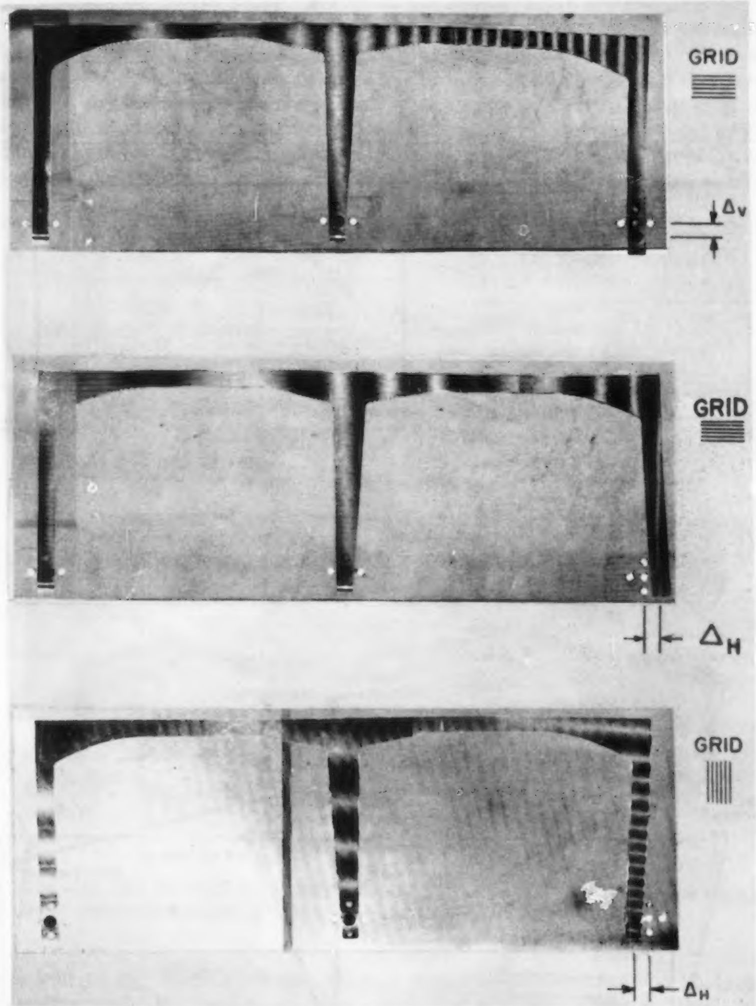

FIG. 5.—MOIRÉ PATTERNS FOR VERTICAL AND HORIZONTAL DISPLACEMENTS AT THE RIGHT HAND SUPPORT OF TWO-BAY FRAME WITH HAUNCHES

and nearly equal vertical displacements in two directions were applied at the end of the short span of the beam. The fringe orders along the beam were plotted and converted to displacements, one fringe corresponding to 1/52 in.

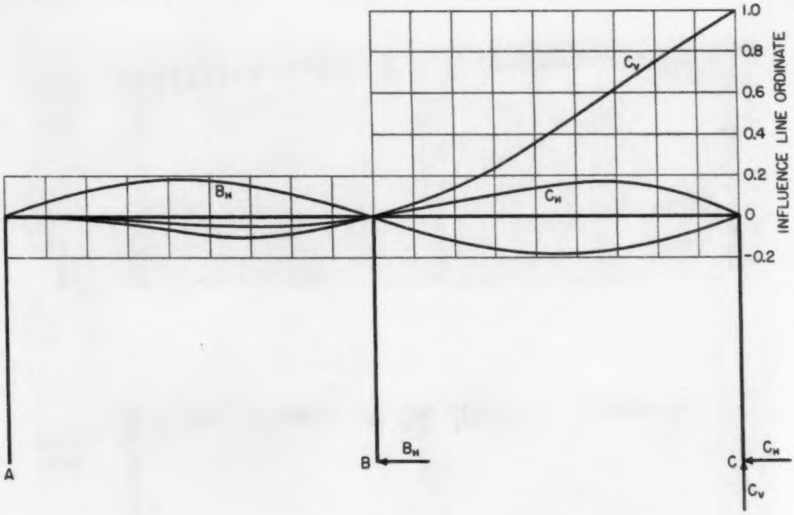

FIG. 6.—INFLUENCE LINES FOR VERTICAL AND HORIZONTAL REACTIONS AT SUPPORT C AND HORIZONTAL REACTION AT SUPPORT B

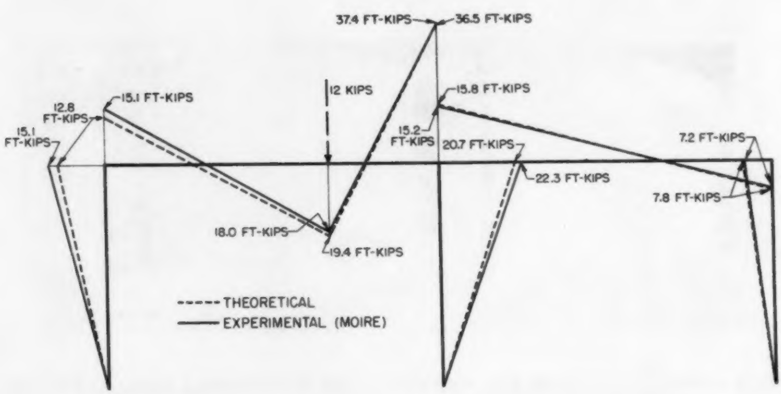

FIG. 7.—MOMENT DIAGRAM FOR TWO-BAY FRAME WITH HAUNCHES

of vertical displacement. These displacements divided by the total traverse of the end of the short span give the ordinates of the influence line for the reaction at that end. The experimental and theoretical influence lines for this reaction are shown in Fig. 3. The small discrepancy at the center of the large span may be due to some friction at the supports.

Frame with Haunches.—Experimental methods in general gain advantage over the theoretical ones when the geometry of the structure becomes complicated. A two-bay frame with parabolic and tapered haunches and a tapered column was analyzed here for the case of a concentrated vertical load (Fig. 4). Grids of 60 horizontal lines per in. were used as master and model grids. Horizontal and vertical displacements in two opposite directions were applied at the ends of the outer columns. Horizontal displacements were also applied

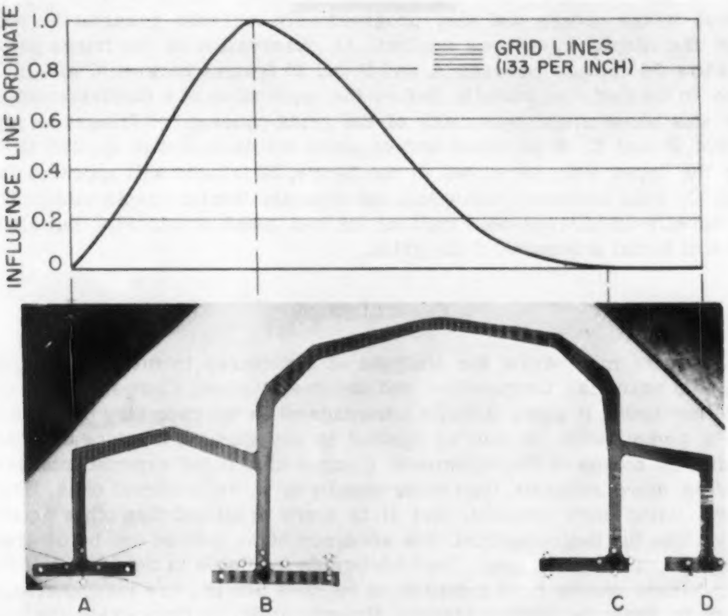

FIG. 8.—MOIRÉ FRINGE PATTERN IN A FRAME SUBJECTED TO A VERTICAL DISPLACEMENT AT ONE OF THE CENTRAL SUPPORTS

at the end of the middle column. Fringe patterns for vertical and horizontal displacements at the end of an outer column are shown in Fig. 5. From such fringe patterns, by averaging effects of opposite displacements and taking advantage of symmetry when applicable, the influence lines of Fig. 6 were obtained. From these influence lines, the moment diagram corresponding to the given vertical load of 12 kips was determined and compared with the theoretical diagram as shown in Fig. 7. Except for one point, the agreement seems to be satisfactory. The discrepancies may be due partly to experimental errors and partly to the fact that the theory did not take into account the true

rigidity of the joints. In structures of the type studied here, the joints are large undeformed areas with very high rigidity.

Three-Bay Frame.—A three-bay frame of complicated geometry like the one shown in Fig. 8 would be exceedingly difficult to analyze theoretically. However, the moiré method lends itself easily to the analysis of such a frame. A model was machined out of a plexiglas plate with a horizontal grid of 133 lines printed on it. The model was mounted on a similarly printed plate. Provision for the application of displacements and rotations at the points of support can be seen in Fig. 7. Fig. 8 shows the fringe pattern corresponding to a vertical displacement of support B. From this pattern, the influence line of the vertical reaction of support B is determined and plotted on the same figure. Fringe orders were counted starting from the fixed supports A and C where the displacement is zero. The light bands represent integral fringe orders and they progressively increase towards the column where the displacement was applied. An observation of the fringe patterns will show 34 fringes between A and B and 36 fringes between C and B. This is due to the fact that initially, before the application of a displacement at B, there was some slight mismatch of the grids causing two fringes to appear between B and C. If an equal and opposite displacement is applied at B, by using the upper hole as shown in the figure, 32 fringes will appear between B and C. This indicates that equal and opposite displacements eliminate not only the effects of large deformations (as mentioned previously), but also the effects of initial mismatch of the grids.

CONCLUSIONS

The moiré method for the analysis of structures by means of displacements is a practical, inexpensive, and accurate method. Compared with theoretical methods, it gains definite advantage when the geometry of the structure is complicated. It can be applied to any kind of structure that can be analyzed by means of displacements. Compared to other experimental methods, it is more accurate than other equally or more practical ones, like the methods using wire models, and it is more practical than other accurate methods like the Beggs method. The accuracy of the method can be controlled by the pitch of the grids used. The finest grids available in the form of Artype or Zip-a-tone sheets have a density of 60 lines per in., are inexpensive, and as easy to apply as postage stamps. Unfortunately, no finer grids are available in this form, and whenever higher accuracy is required, finer grids must be printed directly on plastic.

ACKNOWLEDGMENTS

The present paper is based on research sponsored by the National Science Foundation and is also supported by the Armour Research Foundation. The support received from these organizations is gratefully acknowledged.

Journal of the

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

DISCUSSION

Note.—This paper is a part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 12, December, 1960.

INSTALLATION AND TIGHTENING OF HIGH STRENGTH BOLTS[a]

Closure by E. F. Ball and J. J. Higgins

E. F. BALL,[1] F. ASCE and J. J. HIGGINS.[2]—The authors wish to express their appreciation for the interesting discussions of their paper by Messrs. Zweig, Zar, Munse.

It was noted by both Messrs. Zweig and Zar that the "turn-of-nut" method, as described in the paper, does not seem to control the tightening of the fitting-up bolts as rigidly as that of the other bolts. It has been found, however, in field testing completed joints with calibrated torque wrenches, that the tension in the bolts used for fitting up is not very different from the tension in the other bolts. This was also found to be the case in laboratory tests made at Lehigh University.[3]

Bethlehem's experience in high strength bolting since July, 1959, has demonstrated that better results can be obtained by using a large percentage of the bolts in a joint for fitting up purposes, even up to 100%. In most cases, proper fit-up can be obtained without tensioning the bolts beyond the snugging-up condition. Final tightening is then accomplished by giving all of the bolts one-half or three-quarters of a turn, working progressively away from the fixed or rigid ends to the free edges.

Mr. Zar points out the desirability of a thorough inspection ritual. The authors agree with him on this and also agree that the calibrated torque wrench method of checking tension in the bolts is the only practical known method for field inspection.

Mr. Munse asks if the authors have any records regarding the capacity of bolts after retightening several times. Lehigh University made some tests,[4] in September, 1959, and concluded that bolts, properly installed, could be reused as often as five times without reaching ultimate, and still have a factor against rupture of approximately 2. In practice, bolts are usually tightened only once, but in rare cases they might be tightened two or possibly three times.

---

[a] March, 1959, by E. F. Ball and J. J. Higgins.

[1] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.

[2] Supervisor of Erection, Tool Houses & Safety, Bethlehem Steel Co., Bethlehem, Pa.

[3] "Static Tension Tests of Compact Bolted Joints," by Robert T. Foreman and John L. Rumpf, Proceedings, ASCE, Vol. 86, No. ST 6, June, 1960.

[4] "Calibration and Installation of High Strength Bolts," by Robert A. Bendigo and John L. Rumpf, Fritz Engrg. Lab. Report No. 271.7, September, 1959.

STABILITY CONSIDERATIONS IN THE DESIGN OF STEEL COLUMNS[a]

Closure by Charles E. Massonnet

CHARLES E. MASSONNET,[1] F. ASCE and MEMBER I. ABSE.—The author thanks Messrs. Eremin and Galambos for their appreciation of his paper and interesting contributions.

There seems to be a misunderstanding in the discussion by Mr. Eremin of the author's Eq. 17. It should be emphasized that the end moments M_1 and M_2 are considered positive if the curvature of the member keeps the same sign along the length of the bar. In the opposite case, M_1 is considered positive and M_2 negative with $|M_1| \geq |M_2|$. In this way, no discontinuity appears in the author's equation.

For what regards the coefficient of safety, it depends entirely on the permissible stresses, that is, on the specification adopted, and the author cannot therefore, propose any definite figure.

The author agrees with the remark of Mr. Galambos that the secant formula may be unsafe in certain regions. The opinion of the author on this point is obviously applicable only to members free of residual stresses. The author may observe, in this connection, that his Eq. 17 as well as the charts presented by Messrs. Galambos and Ketter apply especially for rolled bars containing the usual amount of residual stresses. They give a too large safety for stress-relieved bars and could be unsafe for bars fabricated by welding, as is demonstrated by recent tests.[2,3]

The comparison between the results of author's equations and the elaborate elasto-plastic analysis of the lateral-torsional buckling made recently by Mr. Galambos is most interesting and confirms the practical value of both approaches. The indicated discrepancies between Eq. 17 and the more rigorous analysis are inavoidable, because in Eq. 17 the same numerical coefficients must represent two completely different behaviors, namely, buckling, by bending in the plane of applied moments and lateral-torsional buckling.

Finally, the author agrees with Mr. Galambos' last remark concerning the fact that the most serious consequence of lateral-torsional buckling is that it may reduce the rotation-capacity of a column. This was observed experimentally in the buckling tests by F. Campus and the author.

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[a] September, 1959, by Charles E. Massonnet.

[1] Prof., Engr. Mech., Univ. of Liège, Liège, Belgium.

[2] "The Influence of Residual Stresses on the Buckling of Steel Members," (in german) by B. Thürlimann, Schweizer Archives, December, 1957.

[3] "Improvement of the Buckling Load of I Steel Members by Introducing Adequate Residual Stresses," (in french) by H. Louis, C. Massonnet, P. Guiaux, P. Hallet, and G. Kayser, Revue de la Soudure, Belgium, (publication pending).

LESSONS OF COLLAPSE OF VANCOUVER 2ND NARROWS BRIDGE[a]

Closure by A. Hrennikoff

A. HRENNIKOFF,[1] F. ASCE.—The discussion of Mr. Kuang-Han Chu questions the author's interpretation of the column formulas. Mr. Kuang attaches a different significance than the author to the term l in the formula for the pin-ended members of the Standard Specifications of the American Association of State Highway Officials (AASHO):

$$f = 15,000 - \frac{1}{3}\left(\frac{l}{r}\right)^2 \quad \#/\text{in.}^2 \quad \ldots\ldots\ldots\ldots \quad (1)$$

The specification clearly states that in this formula, l signifies the length of the pin-ended member, as the author has used it, while according to Mr. Kuang, l should be taken as the length of the member divided by 0.85. For justification of his interpretation, Mr. Kuang refers to the Final Report of the Special Committee on Steel Column Research in Vol. 98, 1933, of Transactions, ASCE, and this reference requires some clarification.

The tests performed by the Committee involved only columns provided with riveted ends and the results of these tests led to the following formula for such members:

$$f = 15,000 - \frac{1}{4}\left(\frac{l}{r}\right)^2 \quad \#/\text{in.}^2, \quad \ldots\ldots\ldots \quad (2)$$

in which l is the length of the member. Because no tests with pin-ended members had been carried out, the Committee had to use judgment for devising an appropriate formula for the members of this kind. Realizing that a pin-ended column is more susceptable to buckling than one of the same length but having riveted ends, the Committee felt that the formula for the pinned ends may be of the same type as for the riveted ends but having a somewhat greater numerical coefficient before $\left(\frac{l}{r}\right)^2$ in the term subtracted from 15,000. This higher coefficient was found, largely by judgment, using $\frac{l}{0.85}$ instead of l in the above formula, that is,

$$\frac{1}{4}\left[\frac{\left(\frac{l}{0.85}\right)}{r}\right]^2 = \frac{1}{2.89}\left(\frac{l}{r}\right)^2 \sim \frac{1}{3}\left(\frac{l}{r}\right)^2 \quad \ldots\ldots\ldots \quad (3)$$

In other words, the Committee felt that a pin-ended column of the length l was as strong as a column with riveted ends of the length $\frac{l}{0.85}$. This reasoning of

[a] December, 1959, by A. Hrennikoff.

[1] Prof., Civ. Engrg., Univ. of British Columbia, Vancouver, B. C., Canada.

the Committee, consistent with the author's interpretation, was apparently misconstrued by the discusser.

Mr. Donnemann attributes the failure of the grillage stringers to their lack of lateral rigidity. The writer has no quarrel with this conclusion since to him the lack of lateral rigidity in a strongly compressed member means much the same thing as the longitudinal instability, but he disagrees with the reviewer's diagram and computations.

Mr. Donnemann's sketch of the falsework column in Fig. 1, under the title "Rotation," shows moment M_A at its top end. The writer does not see how this moment could have been developed in the presence of a hinge connection between the column and the truss. The hinge involved two cylindrical surfaces of 2 ft-6 in. and 2ft-8 in. radii on which the column rocked in relation to the truss. A key prevented relative linear displacement of the two members. Under these conditions the reaction of the truss on the column was forced to act centrally at all times in relation to the top end of the column.

Mr. Donnemann's picture of the deformation of the column and grillage in the same figure under the title "Translation" violates statics. The only force which he shows in this figure is the horizontal force Q. Because this is an internal force between the column base and the grillage, its direction as shown must be judged, in view of the indicated manner of deformation of the grillage stringers, as that of the force acting on the grillage, which is opposite to the one acting on the column. With these considerations in mind, the free-body diagram of forces acting on the column, implied in this figure, would be as shown in Fig. 13, which is manifestly impossible in view of all moments acting in the same direction.

With a small horizontal displacement of the top of the column caused by temperature and indicated in a grossly exaggerated manner in Fig. 14, the thrust in the column must follow one of the lines AB, AB_1 or AB_2. Which of these three is the correct one and how great is the eccentricity e, is determined, as in all statically indeterminate conditions, by the deformations involved, which in this case consist of the flexural deformation of the column, the compression of the two plywood pads, and the flexure of the webs and flanges of the stringers. The structure below the plywood may be considered absolutely rigid.

The thrust cannot take the line AB, since the absence of eccentricity at the bottom would result in a central compression of all members involved and thus would not allow for a tilt of the column base. Nor could the thrust follow the line AB_2 because the inclinations of the column base as determined by the flexure of the column on the one hand, and by the compression of the plywood, on the other, would be incompatible. Consequently, the thrust line must follow the direction AB_1, and the amount of eccentricity, e, must be such as to make the angular deformations of the column and of the plywood packing compatible, as shown pictorially in Fig. 15. (The compression of the bottom plywood pad is left out in this figure for simplicity of presentation).

It is not wrong to use the vertical and horizontal components N and Q in place of the thrust P, but the fact that these components keep a certain ratio between them, implied in the magnitude of the eccentricity e must not be forgotten. By the way, the direction of Q under these conditions comes out opposite to the one indicated by the discusser.

The fact that there is an eccentricity, e, at the bottom of the column and in the supporting structure does not mean that each stringer receives a thrust with the same eccentricity e, but it means that the thrust is divided unequally

between the stringers, the extreme stringer one the side of the eccentricity receiving the greatest share of the load and through that compressing the plywood the most.

Ignoring any differences and imperfections in stringers and considering the top and the bottom plywood pads identical, one can realize that the upper and the lower halves of the stringers must bend in a similar manner in order to keep the work of their deformation at a minimum. The stringer thrusts then

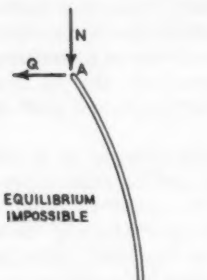

FIG. 13

FIG. 14

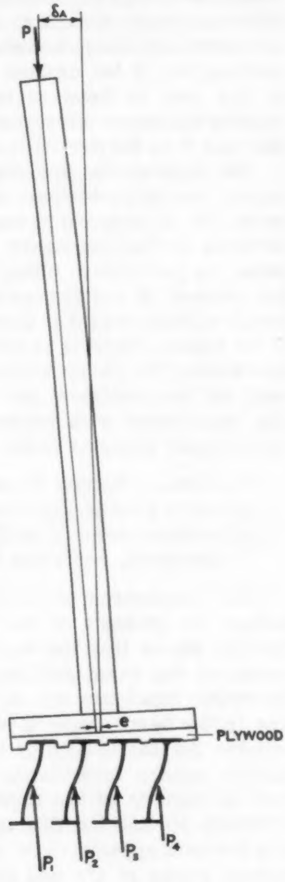

FIG. 15

must be parallel to the column thrust P and must pass through the centers of their deformed webs (see Fig. 15) while their magnitudes P_1, P_2 and so forth adjust in a proper relation to e.

Erroneous results naturally follow if the conditions of statics are used without realizing the true structural action of the column and its support.

Mr. Donnemann assumes an arbitrary eccentricity of the thrust at the top of the stringer and only half as great an eccentricity at its bottom. In the first place, in the conditions assumed by the discusser the beam flanges would cease to be parallel and this would immediately require compensating deformations on the part of the plywood pads which would shift the eccentricity towards the equality of the top and bottom moments. Furthermore, the assumed eccentricity results in a plastic stress at the top of the web. (By the way,) it seems as though the discusser has forgotten to add on the direct compression stress). From this Mr. Donnemann concludes that the stringer is overstressed. Actually, however, the conclusion should be that the discusser's assumption of the amount of eccentricity is unrealistic because a tendency of the web to bend in the direction of assumed eccentricity, that is, in a counterclockwise direction, in Fig. 2, would immediately lead to a shift of the load P in the direction of a reduced eccentricity.

Mr. Donnemann discusses also the two principal points brought up in the paper: the imperfections of the shape of rolled sections and the role of plywood. He is inclined to attach no special significance to the observed imperfections of the stringers, in general, and to the initial curvature of their webs, in particular, citing some figures in support of his opinion. Actually the amount of initial curvature in a column cannot be judged too large or too small without regard to the column formula used for the design of the member. If the column formula is very conservative then even a large curvature is not excessive. The author wrote his conclusions having in mind the column formulas used on this continent and more particularly the ASSHO and CSA formulas. His experiment with compression of fixed ended 36 WF 160 beams described in the paper resulted in the following values:

Theoretical failure stress by Euler's formula = f_y =	33 kips/sq in.
Actual failure stress from experiment	18.23 kips/sq in.
Allowable stress by AASHO, without extra erection allowance, and using l = 16 1/8 in.	12.58 kips/sq in.

The comparison of the first two figures indicates that the imperfections reduce the strength of the beam nearly in half, and the ratio of the last two figures shows that the factor of safety in using the AASHO formula for the design of the beam web in question is reduced to as little as 1.45. From this the writer concludes that in the conditions of the test the effect of irregularities in the beam shape is very substantial and that the AASHO formula is not suitable for the design of the beam. This puts a question mark on the use of AASHO column formula for the design of beam webs in buckling in general, that is, outside of the conditions of the test, but, of course, does not prove definitely its unsuitability in those regions. A different and stricter compression formula appears to be needed for the design of beam webs at least within certain limits of l/r and the German formula, referred to by the reviewer, may possibly meet the requirements.

Mr. Donnemann also does not think that the plywood pads had any substantial effect on instability of the stringers, basing his belief on a theoretical derivation which, unfortunately, was omitted from his discussion. The writer himself had followed the same line of theoretical approach, but the results were inconclusive, and knowing the complexity of the matter he is unwilling to accept, without proof, the discusser's statement.

It should be noted that the opinions expressed by the author in his original paper are his own and not the ones of the Royal Commission, for which he is

not authorized to speak. His views, however, in no way contradict the findings of the Royal Commission.

The writer wishes to add that in his recently conducted new tests involving buckling of webs of 36 WF 160 beams there has been found some corroboration of the thoughts expressed by him in the paper under discussion. However, detailed description of these tests will have to wait until the results are fully worked out.

The writer is grateful to the discussers for their contributions to the subject of the paper. He regrets, however, the absence of comments on his criticism of the column formula of the Canadian Standards Association within certain limits of the slenderness ratio.

STABILITY CONSIDERATIONS IN THE DESIGN OF STEEL PLATE GIRDERS[a]

Closure by Charles E. Massonnet

CHARLES E. MASSONNET,[3] F. ASCE, Member IABSE —The author agrees with Mr. Eremin's observations and thanks him for his interesting suggestions of undertaking research on steel plate girders with haunched web plates.

[a] January, 1960, by Charles E. Massonnet.
[3] Prof. of Engrg. Mech., Univ. of Liège, Liège, Belgium.

[illegible]

[illegible]

[illegible]

[illegible]

PROPERTIES OF STEEL AND CONCRETE AND THE BEHAVIOR OF STRUCTURES[a]

Discussion by A. Zaslavsky and Paul Zia

A. ZASLAVSKY.[9]—This comprehensive and authoritative review by Winter covers a topical subject of great practical importance. The writer would like to add the following remarks:

1. The term "Strength of Materials" is certainly a misnomer; so are "Resistance of Materials" as used in French, Italian, Russian, and so forth, and "Theory of Strength" as used in German.

2. The author suggests that the latest developments in structural design are characterized by "strength" vs. "stress" rather than by "plastic" vs. "elastic." This is correct; the writer wonders, however, whether (in limit desing) it is not, in the last resort, "strain" vs. "stress," as ultimate strength is actually defined by a limiting deformation (strain) criterion even if expressed in terms of strength (stresses). For instance, the appearance of a plastic hinge in a simple steel beam does not exhaust the beam's carrying capacity, there is still the strain hardening reserve available. But because of the large deflections (strains) involved, the plastic hinge stage is defined as the ultimate strength stage. Another example is the Austrian Code for plastic design of reinforced concrete where the critical moment is defined not as the one causing crushing of the concrete, but rather as the one producing a certain critical limiting compressive strain (or, alternatively, the beginning of the steel's yielding); this is in line with author's remarks.

In this connection one may also define the buckling load as the longitudinal load producing very large transverse deflections when its eccentricity approaches zero.

Finally strain (deformation) could serve as a common criterion for both the critical ("collapse") stage of the structure and its working stage where deflections and cracks are characterized by strain, and allowable working stresses could be expressed by "allowable strains."

3. Referring to the author's Fig. 6, it is a very good example for showing the optimum design possibility $\sigma_1 = \sigma_2 = \sigma_y$ by the plastic method, while in the elastic range: $\sigma_1 = \sigma_2/\cos^2 \sigma_2$ independently of the section ratio A_1/A_2. But the example of the fixed-ended beam is of a different nature, since the elastic bending moment diagram does depend on the section's variation along the beam. It is theoretically possible, by appropriate strengthening of the ends, to achieve an elastic bending moment diagram where the yield stress (or working stress) will appear simultaneously at the extreme fibers of both ends and of the midspan sections. In such a beam, no moment redistribution takes place in the

[a] February, 1960, by George Winter.

[9] Senior Lecturer, Technion, Israel Inst. of Tech., Haifa, Israel.

plastic range since all the plastic hinges will from simultaneously and in this case results by both methods will differ by the shape factor only (just as in a statically determinate beam).

4. Under the heading "Steel: Residual Stresses," the statement introducing Eq. 3 is not quite clear, because buckling may also take place under (average) stresses well below the proportional limit.

5. Under the heading "Steel: Plastic Design," (in the 10th line under Fig. 6) it might be clearer if the term "static moment" were replaced by "bending moment."

PAUL ZIA,[10] M. ASCE.—Mr. Winter has made an excellent presentation of the recent findings on performance of steel and concrete and their effects on structural behavior. On the subject of failure of concrete under combined stresses, the author made references to the efforts of Cowan (20), McHenry and Karni (21), Bresler and Pister (22) in searching for simplified failure criteria. In this discussion, the writer would like to present a modification to Cowan's criterion which he proposed in his recent study of torsion of prestressed concrete members. (The result of this study is to be published by the ACI Journal).

The modified Cowan's criterion is shown in Fig. 12 in the form of a failure envelope. The construction of this envelope depends on two distinct properties of concrete, namely, the compressive strength f'_c and the torsional strength τ_o. It is seen that this envelope is, in essence, a close approximation of Mohr's generalized failure envelope. Fig. 13 shows the considerable difference among the various strength theories expressed in the form of interaction curves. To test the validity of the proposed criterion, the test data from various sources are plotted in Fig. 14 in terms of the principal stresses. These test data are seen to have very reasonable correlation with the theory, considering the divergence of the types of test specimens. Fig. 15 is a design aid, prepared according to the proposed criterion, from which one may readily determine the apparent torsional strength of concrete subjected to various magnitudes of compressive stress.

Regarding the tensile strength of concrete, the writer agrees with Mr. Winter that it is not a constant fraction of the cylinder strength. The writer, however, suggests the following relationship:

$$f'_t = 0.68 \left(f'_c\right)^{3/4} \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

This relationship is based upon the extensive test results of H. F. Gonnerman and E. C. Shuman[11] and is shown in Fig. 16.

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[10] Asst. Prof. Civ. Engrg., Univ. of Florida, Gainesville, Fla.

[11] "Compression, Flexure and Tension Tests of Plain Concrete," by H. F. Gonnerman and E. C. Shuman, Proceedings, ASTM, Vol. 28, Part II, 1928, p. 527.

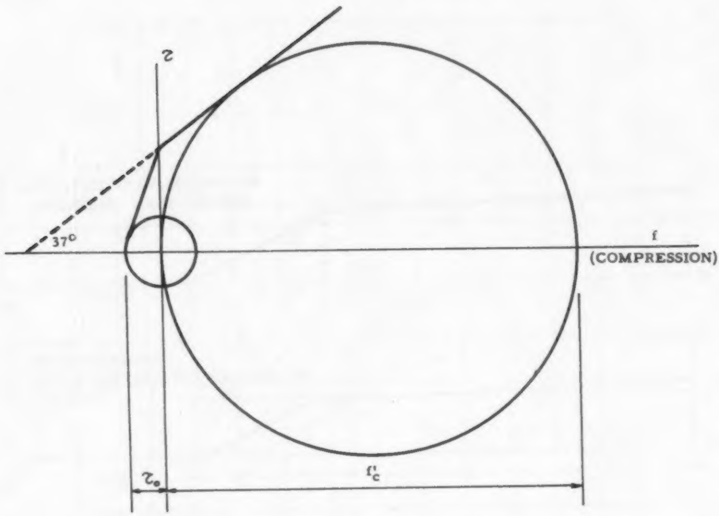

FIG. 12.—MODIFIED COWAN'S THEORY OF FAILURE

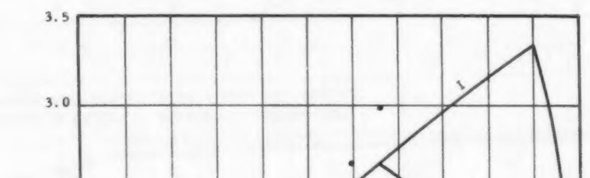

FIG. 13.—INTERACTION CURVES FOR τ_o = 0.10 f'_c

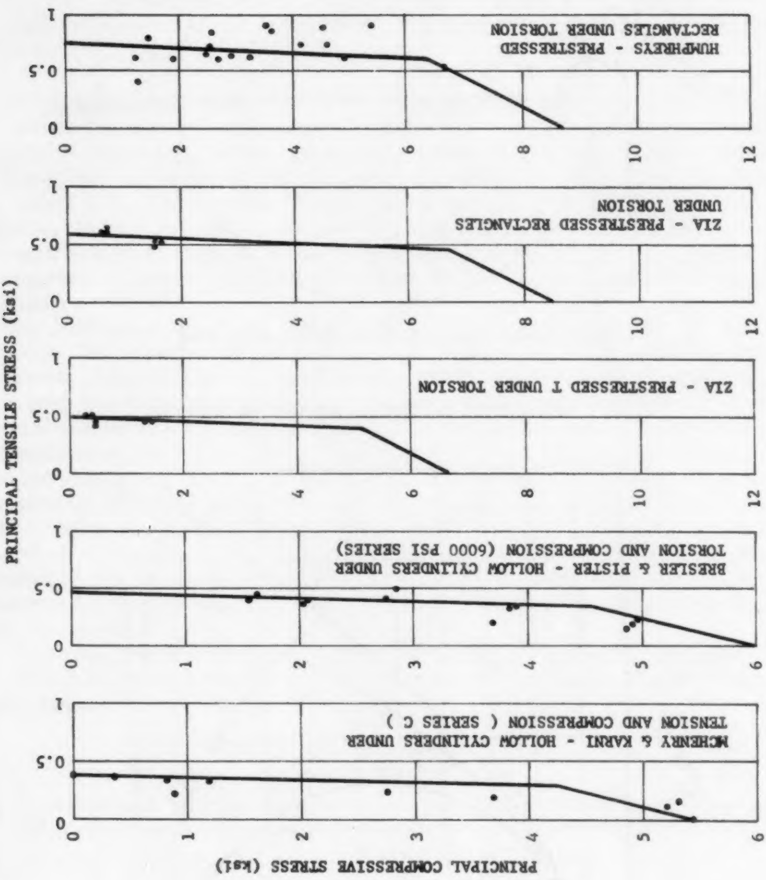

FIG. 14.—COMPARISON OF MODIFIED COWAN'S THEORY WITH EXPERIMENTAL DATA

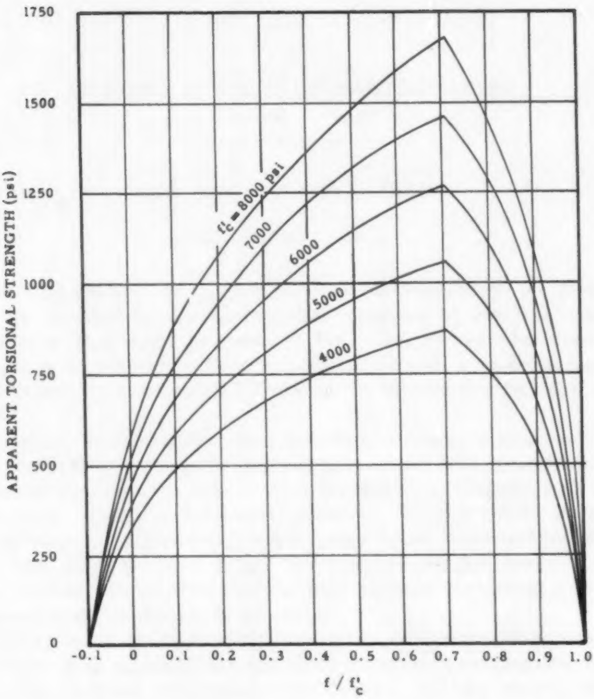

FIG. 15.—APPARENT TORSIONAL STRENGTH VERSUS f/f'_c

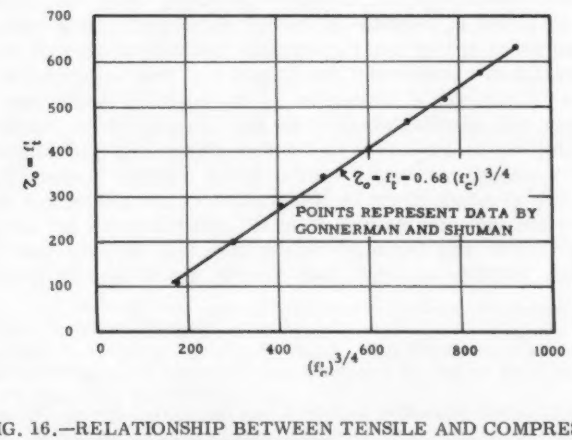

FIG. 16.—RELATIONSHIP BETWEEN TENSILE AND COMPRESSIVE STRENGTH OF CONCRETE

DYNAMIC EFFECTS OF EARTHQUAKES[a]

Discussion by John A. Blume

JOHN A. BLUME,[24] F. ASCE.—A useful summary of basic dynamic principles as applied to the earthquake response of idealized single-mass elastic systems has been presented. Mr. Clough has also considered the modal response of elastic multi-story buildings and, with the aid of the work of Penzien in inelastic response,[25] has drawn interesting comparisons to the SEAOC code.[5]

It is gratifying to the writer, as a member of the committee which drafted this code, to find the author in general agreement with the document and its various simplified approaches to the complex earthquake problem. Some word of caution might be indicated, however, to prevent those not familiar with all aspects of engineering seismology from interpreting the paper to mean that the SEAOC code might be a substitute for dynamic analysis of special or unusual structures such as high slender buildings, or buildings of marked asymmetry in plan or in elevation.

The SEAOC code is "intended to provide minimum standards as design criteria toward making buildings and other structures earthquake-resistive."[5] This carefully worded quotation from Section 2312(a) should not be overlooked. Moreover, in the introduction to the code document, the committee reports: "like any progressive building code, this is an interim code. The committee realizes there is much work to be done as the results of research and further study become available."

The SEAOC code is an advanced document which, although containing some approximations and compromise values is modern, is based on a great deal of study and structural-dynamic considerations, and is in terms and procedures which introduce new and significant parameters to earthquake codes. The main committee consisted of 16 engineers. Many others participated in sub-committees, study groups, and as code consultants. Six members of the main committee had previously served as members of the joint committee of the San Francisco Section, ASCE and the Structural Engineers Association of Northern California which proposed[26] several concepts that are now incorporated in the SEAOC Code. Thus it can be said that committee efforts leading to this present document were expended intermittently over 10 yr with thousands of man hours of volunteer time. In addition, independent but

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[a] April, 1960, by R. W. Clough.

[24] Pres., John A. Blume & Associates, Engrs., San Francisco, Calif.

[25] "Dynamic Response of Elasto-Plastic Frames," by Joseph Penzien, Proceedings, ASCE, Vol. 86, No. ST 7, July, 1960.

[26] "Lateral Forces of Earthquake and Wind," by Anderson, Blume, Degenkolb, Hammill, Knapik, Marehand, Powers, Rinne, Sedgwick, and Sjoberg, Transactions, ASCE, Vol. 117, 1952.

related research and analysis of the energy absorbtion value of buildings has been conducted.[27] It is perhaps no coincidence that the author finds some similarity to his dynamic considerations in view of the amount of effort, earthquake experience, and judgment applied to the CEAC Code.

The data on the Alexander Building studies to which the author refers, as well as the structure's properties, are available.[28] In this connection, it must be noted that the weights and periods of the building, as shown in Fig. 6, are not in complete agreement with those determined for the prototype. The weights in kips, starting at the top level, were found to be:[28]

1.550, 0.749, 0.809, 0.850, 0.869, 0.841, 0.841, 0.847, 0.856, 0.865, 0.863, 0.886, 0.918, 0.931, and 1.173.

The periods in seconds are as follows:

Mode:	1st	2nd	3rd	4th
Parallel to Montgomery Street	1.25	0.41	0.24	0.17
Parallel to Bush Street	1.33	0.45	0.26	0.19

Although the differences are not great, they should be explained in view of the importance of this structure as a "guinea pig." Perhaps the writer has treated the building as a "shear" building rather than one that has flexural and ground rotation participation as well as shear deformation. Additional references to the structure would also be of interest since they consider, among other items, the actual response of the building to an earthquake.[27,29]

Other approaches[2,27,30,31,32] to inelastic behavior and design lead to results similar to those reported by Mr. Clough, although conditions can vary widely between buildings and between types of buildings. A big step toward obtaining agreement in the SEAOC Committee came when the concept of various types of buildings having different energy capacities and ductility values (even though designed for the same lateral forces) was introduced. This led to the variable "K" values in the code and introduced the parameters of energy absorbtion and reserve frame ductility in a new manner.

It would be possible, of course, to question certain expressions in the code from an academic viewpoint. A completely rigorous approach to some of the parameters might, and did, result in somewhat different terms. Such matters were fully recognized by the SEAOC Committee but in view of the general problem, the need for simplicity, limits to the amount of acceptable change from existing documents, and compromise measures, the terms and values

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27 "Structural Dynamics in Earthquake-Resistant Design," by John A. Blume, Proceedings, ASCE, Vol. 84, No. ST 4, July, 1958, and discussions ending September, 1959.

28 "Period Determinations and Other Earthquake Studies of a Fifteen-Story Building," by John A. Blume, Proceedings, World Conf. on Earthquake Engrg., San Francisco, Calif., Chapter 11, June, 1956.

29 "A Comparison of Theoretical and Experimental Determinations of Building Response to Earthquakes," by D. E. Hudson, 2nd World Conf. on Earthquake Engrg., Tokyo, Japan, July, 1960.

30 "Effect of Inelastic Behavior on the Response of Simple Systems to Earthquake Motions," by A. S. Veletsos, and N. M. Newmark, Proceedings, 2nd World Conf. on Earthquake Engrg., Tokyo, Japan, July, 1960.

31 "A Reserve Energy Technique for the Design and Rating of Structures in the Inelastic Range," by John A. Blume, Proceedings, 2nd World Conf. on Earthquake Engrg., Tokyo, Japan, July, 1960.

32 "Energy Consumption by Structures, in Strong-Motion Earthquakes," by G. V. Berg and S. S. Thomaides, Univ. of Michigan Report 2881-2-P, March, 1960.

were developed as shown.[5] It has been said that the camel is a horse designed by a committee. However, the camel is a very practical animal in his natural environment.

It should be noted that (1) codes must be simple to be accepted and to be used; (2) earthquake codes cannot wait for precise or scientific methods; (3) codes should not be considered as a complete answer to the design of all structures, especially these of a special or unique nature; (4) a group of dedicated men has produced a workable document with at least general consideration of structural-dynamic behavior; and (5) the SEAOC code, now adopted by the International Building Officials Conference as well as the City and the County of Los Angeles, represents, in the opinion of a great many engineers, a logical and practical minimum earthquake design procedure for typical structures. However, the need for sound engineering judgement for the effective use of this or any other code should not be overlooked.

REVIEW OF RESEARCH ON COMPOSITE STEEL—CONCRETE BEAMS[a]

Discussion by J. C. Chapman

J. C. CHAPMAN.[4]—Congratulations are in order for this excellent and valuable review. This paper will be welcomed by all practicing engineers and research workers, since all too few papers of this kind are written.

There is always a tendency to plan a research program without taking complete account of all the related investigations which have been made, largely because of the very considerable effort which is required to do so. Viest has now made the effort for us, and we should all be very grateful.

The list in Table 6 brings the record up to date in respect to tests which have, so far, been made at Imperial College. Non-destructive tests have also been made on two composite building structures. The work is continuing.

[a] June, 1960, by I. M. Viest.

[4] Civ. Engrg., Dept., Imperial College, England.

TABLE 6.—COMPOSITE BEAM TESTS AT IMPERIAL COLLEGE

Description	No. of Specimens	Date	Reference[a,b]
(a) Full Scale Beam Tests (6.in. slab)			
12 x 6 I section with bottom flange plate. 7/8 in. welded studs with heads.	1	1957	1
Cased castellated beam with bottom flange plate. Welded studs without heads.	1	1957	-
12 x 6 I section. Welded tee connectors with hoops.	1	1960	-
12 x 6 I section. 3/4 in. welded studs with heads.	5	1960	-
12 x 6 I section. 1/2 in. welded studs with heads.	1	1960	-
12 x 6 I section. 3/4 in. welded studs with right-angle bend.	1	1960	-
(b) Full Scale Push-Out Tests			
7/8 in. welded studs with heads. 1, 2 and 3 pairs per flange.	8	1956	1
7/8 in. welded studs without heads. 1, 2 and 3 pairs per flange.	6	1956	1
3/4 in. welded studs with heads 1 pair per flange.	3	1960	-
Welded tee connectors with hoops, one connector per flange.	1	1960	-
3/4 in. welded studs with right-angle bend. One pair per flange.	2	1960	-
(c) Small Scale Beam Tests			
2 in. x 1 in. I section cased and uncased with and without 1/8 in. diameter welded studs. 7/8 in. slab.	6	1957	2
4 in. x 1 3/4 in. I sections. 1/4 in. and 3/16 in. welded studs with heads and right-angle bends. 1 1/2 in. slab.	10	1959	2
4 in. x 1 3/4 in. I sections. 3/16 in. welded studs with heads and right-angle bends. 1 1/2 in slab.	11	1960	-
(d) Small Scale Push-Out Tests			
7/64 in. welded studs with heads. 1, 2 and 3 pairs per flange. 3/4 in. slab.	8	1957	2
1/4 in. and 3/16 in. welded studs with heads, with right-angle bends and with inclined bends. In pairs and in tandem. 1 1/2 in. slab.	27	1959	2
3/16 in. diameter studs with heads and right-angle bends.	4	1960	-

[a] "Behaviour of building frames of composite construction," by K. C. F. Wong, Ph.D. Thesis, University of London, 1957.

[b] "Interaction between steel beams and a concrete floor slab," by A. O. Adekola, Ph. D. Thesis, University of London, 1959.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 2270 is identified as 2270(ST9) which indicates that the paper is contained in the ninth issue of the Journal of the Structural Division during 1959.

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DECEMBER: 2271(HY12)[c], 2272(CP2), 2273(HW4), 2274(HW4), 2275(HW4), 2276(HW4), 2277(HW4), 2278 (HW4), 2279(HW4), 2280(HW4), 2281(IR4), 2282(IR4), 2283(IR4), 2284(IR4), 2285(PO6), 2286(PO6), 2287 (PO6), 2288(PO6), 2289(PO6), 2290(PO6), 2291(PO6), 2292(SM6), 2293(SM6), 2294(SM6), 2295(SM6), 2296 (SM6), 2297(WW4), 2298(WW4), 2299(WW4), 2300(WW4), 2301(WW4), 2302(WW4), 2303(WW4), 2304(HW4), 2305(ST10), 2306(CP2), 2307(CP2), 2308(ST10), 2309(CP2), 2310(HY12), 2311(HY12), 2312(PO6), 2313(PO6), 2314(ST10), 2315(HY12), 2316(HY12), 2317(HY12), 2318(WW4), 2319(SM6), 2320(SM6), 2321(ST10), 2322 (ST10), 2323(HW4)[c]. 2324(CP2)[c], 2325(SM6)[c], 2326(WW4)[c], 2327(IR4)[c], 2328(PO6)[c], 2329(ST10)[c], 2330 (CP2).

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c. Discussion of several papers, grouped by divisions.

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PART 2

DECEMBER 1960 — 44
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NO. ST12
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NEWS OF THE STRUCTURAL DIVISION OF ASCE

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

JOURNAL OF THE STRUCTURAL DIVISION
PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

DIVISION ACTIVITIES
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

NEWS

December, 1960

Contents

SIXTH CONGRESS OF THE IABSE

The International Association for Bridge and Structural Engineering held its Sixth Congress in Stockholm, Sweden from June 27 to July 1, 1960. Frank Baron, Professor of Civil Engineering at the University of California, represented the Structural Division at this Congress. This article is an abstract of his report on this Congress to the Executive Committee of the Structural Division.

The attendance at the Sixth Congress consisted of about 800 members of which 40 were American. Thirty-eight nations were represented at the Congress of which the United States had the sixth largest delegation.

Six Working Sessions were held at the Royal Institute of Technology. The theme of each session was discussed and summarized by the general reporter of the session. The reporter and other speakers spoke in one of the three official languages of the IABSE; namely, French, German, or English. A United Nations' flavor was obtained by providing earphones at the desk of each member. When desired, each member could tune in to an immediate French, German, or English translation of a paper.

Note.—No. 1960-44 is Part 2 of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 12, December, 1960.

First Working Session. In this session, particular consideration was given to those characteristics of structural materials that are of importance for design purposes. The contributions to this theme were concerned with fatigue under repeated loads, strength under long term loading, stress relaxation, creep under constant load, and shrinkage of concrete.

Particular mention is made of the differences in views which arose concerning the influences of variable stress values on the fatigue lives of structural members. Attention also is called to the contributions dealing with (a) the dynamic effect of live loads on the superstructure of railway bridges, (b) the ultimate bending strength of reinforced concrete members, (c) the shear strength of reinforced concrete beams, (d) the application of the theory of thin shells to the calculation of arch dams, (e) the calculation of stresses and displacements in a cylindrical shell roof by means of a digital computer, and (f) a plastic theory for the design of reinforced concrete slabs.

Second Working Session. This session dealt with two kinds of connections for metal structures; (1) welds, and (2) pretensioned high-strength bolts. The papers on welds dealt with (a) the effects of longitudinal stresses in fillet welds subjected principally to shear forces, (b) influences of the state of stress, temperature and rate of application of stresses on brittle fracture, and (c) problems arising in the execution of welded structures, and more particularly with the question of quality and the controls required in order to achieve and maintain it.

The papers on pretensioned high-strength bolts summarized the main results of research conducted in the United States, Canada, Great Britain, and Germany. Among the matters reviewed were torque-tension relationships, the turn of the nut method for determining suitable joint assemblies, the load at which first slip occurs, and design criteria for bolted assemblies. Apparently, much of the work which had been initiated and completed in the United States was being duplicated in Europe. Further, the recent recommendations of the Research Council on Riveted and Bolted Structural Joints in regard to the design of joints transmitting load by friction and those by bearing were not generally known in Europe.

Important research was reported by Steinhardt of Germany concerning (a) the load at which the first slip of the joint occurs, and (b) the behavior of a bolted joint when the joint is subjected to combined shear and moment.

Third Working Session. Various aspects of the steel skeleton building were discussed in this session. Among these aspects were current trends in analysis and design. Included were papers on the plastic behavior of structural steel and instability considerations of tall building frames. The latter considerations are gaining increasing importance because of the trends in modern construction towards decreases in dead loads. Attention is called to the investigations on frame instability which were carried out at the College of Science and Engineering, Manchester, England, and summarized by W. Merchant and A. H. Salem.

Considerations also were given to recent trends in floor and wall construction in the United States and Canada. The changes which are occurring in the entire conception of fire protection were discussed by several contributors, including those of Switzerland, Germany, and the United States.

This session would not have been complete without due consideration to erection and safety requirements of steel skeleton construction. These were discussed by several contributors. Important distinctions were noted in erection methods as employed in Europe and America. The methods employed in America were reported by Messrs. Pickworth and Rapp.

Fourth Working Session. The theme of this session was new developments in bridge building with particular reference to reinforced and prestressed concrete. The non-homogeneous character of the papers submitted makes it difficult to give any definite indications of present tendencies. Many examples of arch bridges and continuous girders were presented at this session. An important trend that was observed was the increasing use of prestressed concrete construction for long span bridges.

Among the contributions to this session were papers dealing with (a) the construction of prestressed concrete highway bridges in the U.S.S.R., (b) methods of safety analysis, and (c) the dynamic behavior of 20 prestressed bridges.

Fifth Working Session. Attention was given in this session to prefabricated structures of reinforced and prestressed concrete. The contributions to this theme dealt with (a) safety considerations during lifting, (b) stability of structural components, (c) connection methods for prefabricated elements, and (d) redistribution of stresses due to creep. Two contributors discussed various methods of joining precast concrete elements with specific illustrations for single and multi-story buildings. Another contributor dealt with the residual stresses which are induced by creep and shrinkage in structures composed of precast reinforced concrete members.

Sixth Working Session. In this session, free contributions to important new developments in structural engineering were accepted. Consequently, the various papers had little in common except that all described new or proposed constructional features. The papers dealt with atomic power stations, dams, bridges, plate girders, composite construction, and battle decks. Each paper however was a noteworthy contribution because of its scope and the special problems which arose in the unique constructions. Further, the papers as a group were indicative of the large amount of construction that is occurring on a world-wide basis.

EXECUTIVE COMMITTEE

The Executive Committee of the Structural Division for the year beginning October 1960 and ending October 1961 consists of the following members: Emerson J. Ruble, Chairman; Nathan D. Whitman, Jr., Vice-Chairman, Robert D. Dewell and Ted R. Higgins. Elmer K. Timby, Director ASCE, continues as contact member of the Board of Direction to the Structural Division. Ted R. Higgins, Director of Engineering and Research for the American Institute of Steel Construction is the new member of the committee.

The Executive Committee is the administrative body of the Division. Theirs is the responsibility of planning all activities of the Division.

COMMITTEE ON LOADS AND STRESSES

A new Task Committee on the Design of Towers has been formed with J. R. Arena of Sargent and Lundy as Chairman. At the present time, design criteria for towers are left to the discretion of the designer. As a result, structural engineers concerned with tower design have seen many instances of tower failures or acceptance of unsatisfactory designs performed by unqualified designers.

The function of this committee will be to develop recommendations for the design of Metal Transmission and other Towers.

COMMITTEE ON MASONRY AND REINFORCED CONCRETE

The meeting of the Committee on Masonry and Reinforced Concrete in Chicago on Wednesday, September 7, showed near perfect attendance and a number of constructive reports.

The Task Committee on Prestressed Concrete under Chairman Morris Schupack met a day earlier, on Tuesday, September 6, and established its first ten areas of study in the order of priority. This committee will attempt to avoid overlap with other groups working on prestressed concrete. After it establishes that a particular subject needs special study or research, it will either set up a special sub-committee or request the formation of a new committee to cover this particular topic.

Dr. Eivind Hognestad, Chairman of the Task Committee on Shear and Diagonal Tension, reported that in spite of ten years of work in sponsoring and evaluating research there was as yet no clearly defined theory which covered concrete behavior under the action of shear. The committee is developing a report which is on the state of the art as of 1960 in terms suitable for use by consulting engineers. This report will be put out in at least three parts and will cover beams without web reinforcement, beams with stirrups, slabs, and footings. The committee is cooperating closely with the Comité Européen du Béton in the exchange of European and American ideas.

The model tests of different slab systems sponsored by the Committee on Design of Reinforced Concrete Slabs under the chairmanship of Dr. C. P. Seiss will not be completed for another year. Parallel theoretical studies are under way and much of the future work of this committee in correlating data and theory lies ahead. The committee does not expect to have design recommendations available in time for the 1962 ACI Code but does hope to have their work done in time for the following issue of the code.

Laboratory work on slabs is being extended by a near full size test of a flat plate floor in the Portland Cement Association Laboratories. The Army Engineers also plan to test an actual slab building structure for time and creep effects.

A progress report of the Task Committee on Composite Construction, covering the subject of composite construction for buildings was recommended for publication by the Administrative Committee. This committee under the chairmanship of Dr. Ivan M. Viest, has been quite active in developing this phase of its work in addition to having an organized convention session at Washington last October. The committee looks ahead to other types of composite construction including aluminum members and timber construction.

The committee on Limit Design under Dr. Alfred L. Parme is conducting certain theoretical studies with regard to the ductility needed for limit design work and at the same time supervising a sizeable investigation at the University of Illinois. This committee expects to develop a better method of analysis than that represented by the present elastic analysis. Vice-Chairman Hognestad reported on a meeting in Chicago on Monday, September 5, in which plans were discussed for the exchange of research information, and to some extent the correlation of research originating in this country and in Europe. There is much experimental work which needs to be done and it is hoped that by exchanging research data an earlier understanding of limit design will result.

Mr. Albyn Mackintosh, Chairman of the Committee on Reinforced Masonry Design and Practice said that a report of his committee was now out for committee discussion. There are a number of problems in this field where research is desperately needed but money appears to be very scarce.

The Administrative Committee discussed the desirability of some rotation among the members of the control groups (executive committees) for the different task committees. It was agreed that some rotation was desirable, but that in the case of certain long-time projects the changes should not be made arbitrarily without regard to the status of the committee's work.

COLUMN RESEARCH COUNCIL

Column Research Council announces the publication of its "Guide to Design Criteria for Metal Compression Members." This book of 112 pages presents a condensed summary of design criteria based upon recent as well as past research on metal compression members in buildings and bridges. The book is written especially for the engineer who either meets special problems not covered by standard specifications or is himself engaged in the preparation of specifications for such structures. The book is valuable in furnishing a better understanding of behavior of compression elements and specification requirements whether the material is aluminum, steel, or other metals. Areas of interest covered include centrally loaded columns, compression member details, laterally unsupported beams, and beam-columns. Constructional metals under ASTM Specifications are covered, including the new ASTM A36 Structural Carbon Steel.

The price for this 8-1/2" x 11" leatherette bound volume is $5.00 per copy and orders may be placed with the

Secretary
Column Research Council
313 West Engineering
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STRUCTURAL DIVISION JOURNAL

The present editor of the NEWSLETTER, Gerard F. Fox, has been appointed Sessions Program Chairman of the Structural Division for the October 1961 ASCE Convention to be held in New York City. Frank Randall has been appointed editor of the NEWSLETTER. All news items and minutes of committee meetings for the next issue of the NEWSLETTER should be sent to the new editor at the following address:

Mr. Frank Randall, Newsletter Editor
Portland Cement Association
111 West Washington St.
Chicago 2, Illinois

NEW DIRECTORY IS AVAILABLE TO MEMBERS

The 1960 Directory is now available to members on request. The Directory lists the entire membership of the Society, giving the membership grade, position, and mailing address of each. In addition, there is a complete listing of the Honorary Members, past and present, and the Life Members. A useful geographical listing of the members is also included.

It goes without saying that the information contained in the Directory is of value to every member, and every member can obtain this valuable information. To receive your free copy of the Directory simply fill out the coupon below. Prompt delivery depends on prompt return of the coupon.

The Society publishes the membership Directory every other year. The next edition will be issued in 1962.

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